EE 503

Homework #2
Due: October 20, 2005

Pr.1: Show that

$$(\mathrm{sinc}(x-m), \mathrm{sinc}(x-n)) = \delta[m-n].$$

where (f, g) stands for the inner product calculation: $\int_{-\infty}^{\infty} f(x)g(x)dx$ and m,n are integers. (Hint: Apply Fourier transform to the arguments of the inner product).

Pr.2: The sequences $\mathbf{x}$ and $\mathbf{h}$ have the length N. The sequence $\mathbf{y}$ is defined as the circular convolution of $\mathbf{x}$ and $\mathbf{h}$. Write the circular convolution matrix mapping input vector $\mathbf{x}$ to the circular convolution output $\mathbf{y}$.

Show that the vector $\mathbf{e_k}$ with entries $e_k(n) = e^{j\frac{2\pi}{N}nk}$, $0 \leq n \leq N-1$ is an eigenvector of the convolution matrix. Discuss the link between the eigenvectors and the DFT matrix. (To illustrate your results, you can take $N = 4$.)

Pr.3: We have showed in lectures that if the random variable $\mathbf{y}$ is linearly related to $\mathbf{x}$, that is $\mathbf{y} = a\mathbf{x} + b$, then the correlation coefficient $r_{xy} = 1$. In this problem, you are asked to show the reverse argument.

Show that if $r_{xy} = 1$ then $\mathbf{y}$ has to be necessarily in the form $\mathbf{y} = a\mathbf{x} + b$.

Pr.4: Use convolution matrices to find the inverse of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a & 1 & 0 & \dots & 0 \\ 0 & a & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \end{bmatrix}$$

Pr.5: Problem 2.23 from Therrien

Pr.6: Problem 2.27 from Therrien

Pr.7: Problem 2.28 from Therrien

Pr.8: Problem 2.35 from Therrien