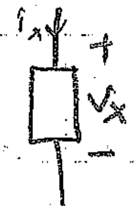
Hilal Güler — EE 202 Spring 2011/12 Sec.03

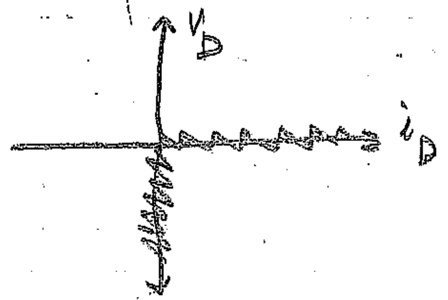
EE 201 Review

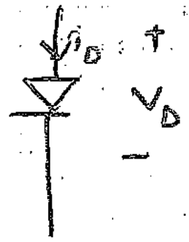
Linear Components : R, L, C ⟶ Terminal equations



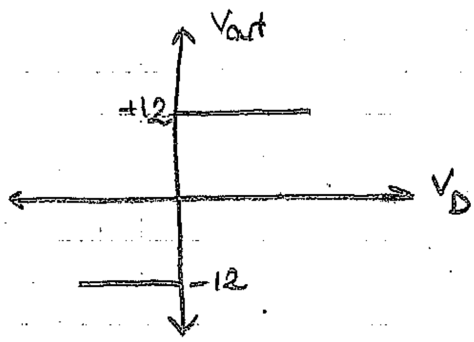
$V_x = f(i_x)$

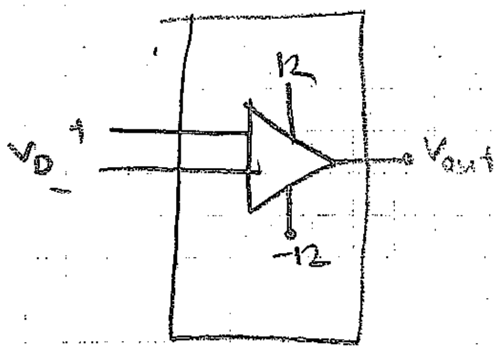
Non-Linear Components:
Diodes, Op-Amp



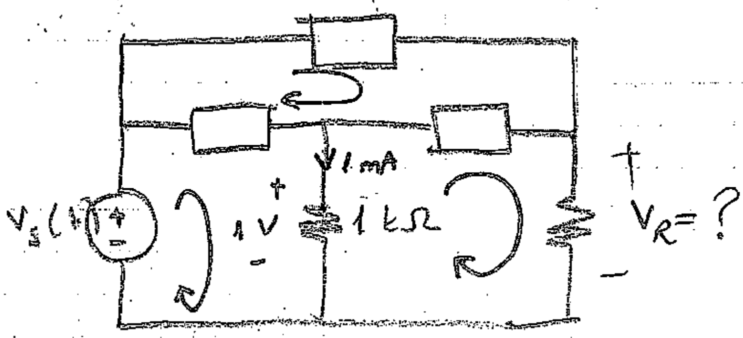


(i, v) char



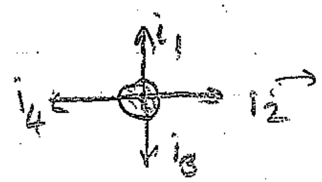


Combination of Components



KVL (∂ Loop) → Conservation of energy

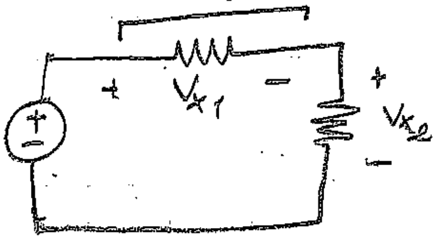
KCL



" of charge

$\sum_{k=1}^{\infty} i_k = 0$ (all exiting currents sum to zero)

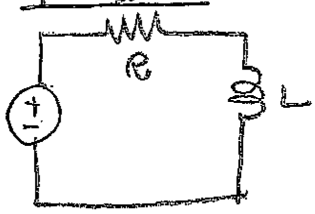
$\Delta W = q\, V_{x1}$ Joules



Systematic Analysis Methods

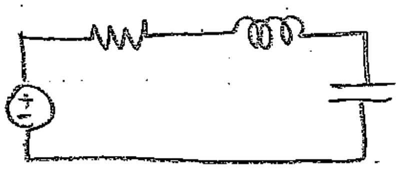
→ Node Analysis → (e_A, e_B, e_C) → Node voltage !
→ Mesh " → (i_1, i_2, i_3) → Mesh currents !
→ Thevenin - Norton

1st Order



RL

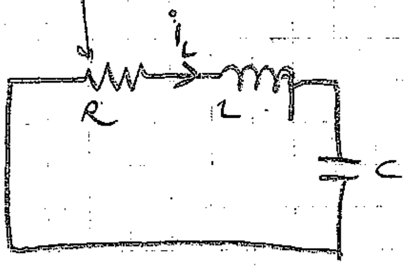
2nd Order Circuit



RLC

→ Step Response
→ Input Response

Zero - State → State : describes "the state" of the circuit
Zero Input Resp.



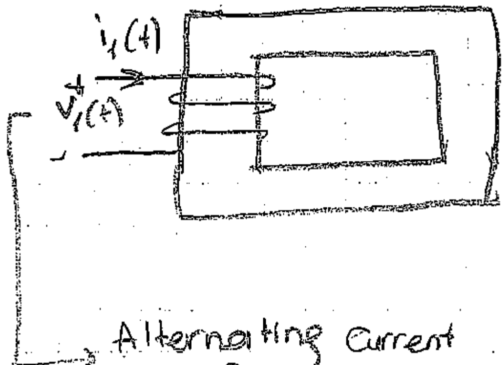
RLC → under - damped
→ over - damped
→ critically - damped

char. eqn : $\lambda^2 + 10\lambda + 20 = 0$

λ: natural freq

$V_C(0^-) = V_0$
$I_L(0^-) = I_0$

Coupled Inductors



→ Alternating Current (AC)

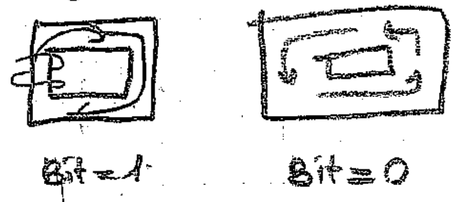
L: Henry (Inductance)

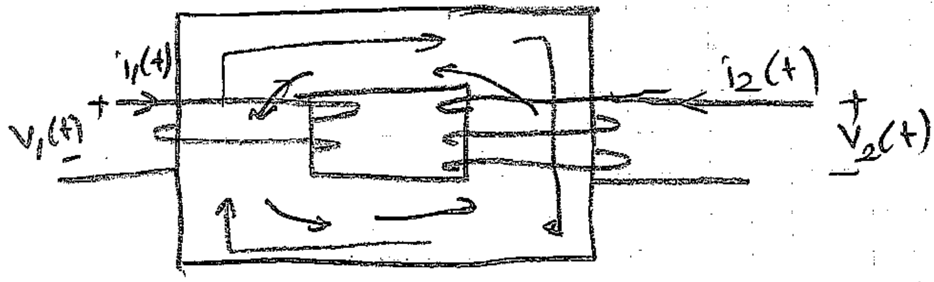
$\phi(t) = L\, i(t)$

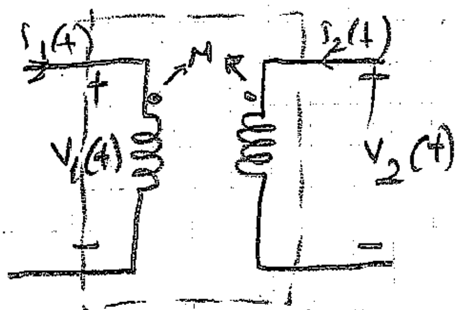
$\frac{d}{dt}$ (Faraday's Law)

$$\boxed{V_L(t) = L\,\frac{d i_L(t)}{dt}}$$

Magnetic Memory:







$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \end{bmatrix}$$

$$\phi_{tot} = \phi_1 + \phi_2$$

$$\phi_{tot} = L_1 i_1(t) + M_2 i_2(t)$$

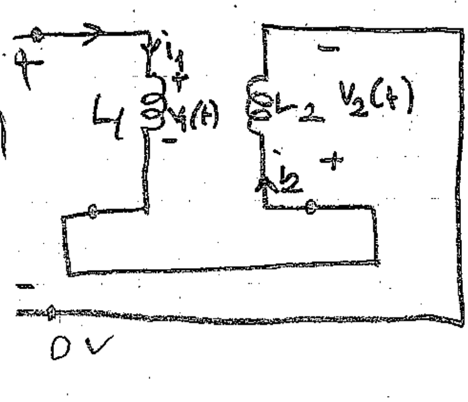
due to flux coupled from 2nd winding

1st winding

$\frac{d}{dt}$

$$\boxed{V_{L_1}(t) = L_1 \frac{d}{dt} i_1(t) + M \frac{d}{dt} i_2(t)}$$

Ex 7



What is the terminal equation of the box?

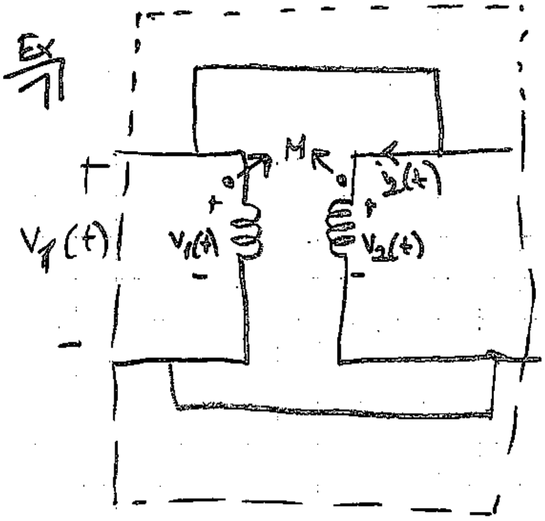
$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \end{bmatrix}$$

$$v(t) = v_1(t) + v_2(t)$$

$$v(t) = v_1(t) + v_2(t) = \begin{bmatrix} L_1+M & L_2+M \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \end{bmatrix} \quad i(t)$$

$$\boxed{v(t) = (L_1 + L_2 + 2M)\frac{d}{dt} i(t)}$$

Ex



"Parallel configuration"

$$v_1(t) = v_2(t) = v(t)$$

$$\boxed{i(t) = i_1(t) + i_2(t)}$$

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \end{bmatrix}$$

$$\underbrace{\frac{d}{dt}}_{D} \underline{i} = \underline{\underline{L}}^{-1} \underline{v} \xrightarrow[D^{-1} \text{ apply}]{} \underline{i} = \underline{D}^{-1} \underline{\underline{L}}^{-1} \underline{v}$$

$$v_1(t) = L_1 \frac{d}{dt} i_1(t) + M \frac{d}{dt} i_2(t)$$

$$\begin{aligned} D^{-1} v_1(t) &= L_1 i_1(t) + M i_2(t) \\ D^{-1} v_2(t) &= M i_1(t) + L_2 i_2(t) \end{aligned} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}^{-1} \begin{bmatrix} D^{-1} v_1(t) \\ D^{-1} v_2(t) \end{bmatrix} = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}}$$

Special case for $\underline{M=0}$

$$\begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} D^{-1} v_1(t) \\ D^{-1} v_2(t) \end{bmatrix} = \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$\frac{1}{L_1} \int_{-\infty}^{t} v_1(\tau)\, d\tau = i_1(t)$$

$v(t)$

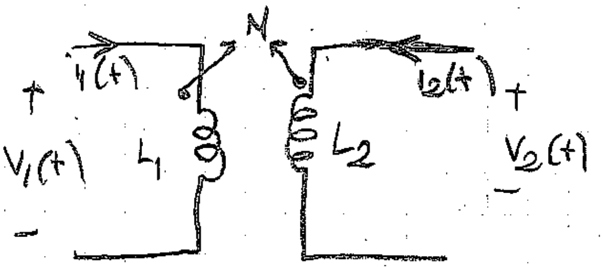
$$i(t) = i_1(t) + i_2(t) = \Gamma_{11} D^{-1} v_1(t) + \Gamma_{12} D^{-1} v_2(t) + \Gamma_{21} D^{-1} v_1(t) + \Gamma_{22} D^{-1} v_2(t)$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}^{-1} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \rightarrow \Gamma$$

$$\boxed{i(t) = \left(\Gamma_{11} + \Gamma_{12} + \Gamma_{21} + \Gamma_{22}\right) D^{-1} v(t)}$$

$$\boxed{i(t) = \frac{1}{L_{eq}} D^{-1} v(t)}$$

Initial Condition Model for Mutual Inductors



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \end{bmatrix}$$

$$\underline{V} \quad \underline{\underline{L}} \quad D \quad \underline{i} \leftarrow \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$i_1(0^-) = I_1$

$i_2(0^-) = I_2$

$$D^{-1}\,\underline{v} = L\;D^{-1}\,D i$$

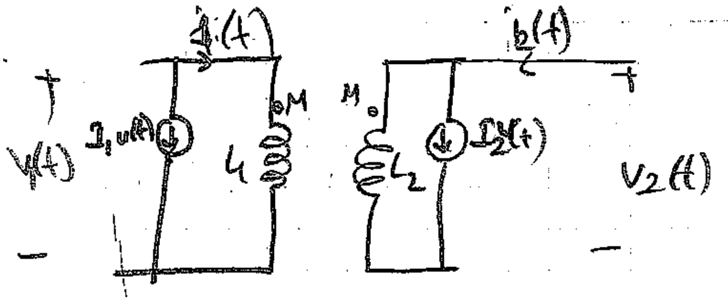
$$\begin{bmatrix} \int_{0^-}^{t} \frac{d}{dt}\, i_1(\tau)\,d\tau \\ \\ \int_{0^-}^{t} \frac{d}{dt}\, i_2(\tau)\,d\tau \end{bmatrix} = \begin{bmatrix} i_1(\tau)\Big|_{\tau=0^-}^{\tau=t} \\ \\ i_2(\tau)\Big|_{\tau=0^-}^{\tau=t} \end{bmatrix} = \begin{bmatrix} i_1(t)-i_1(0^-) \\ \\ i_2(t)-i_2(0^-) \end{bmatrix}$$

$$D^{-1}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} L_1 & m \\ m & L_2 \end{bmatrix}}_{\underline{L}} \begin{bmatrix} i_1(t) - I_1 \\ i_2(t) - I_2 \end{bmatrix}$$

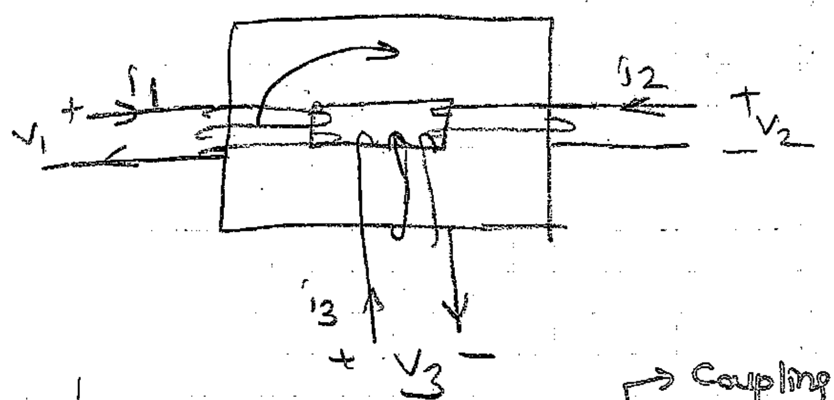
$$L^{-1}\begin{bmatrix} \int_{0^-}^{t} v_1(\tau)\,d\tau \\ \\ \int_{0^-}^{t} v_2(\tau)\,d\tau \end{bmatrix} = \begin{bmatrix} i_1(t) \\ \\ i_2(t) \end{bmatrix} - \begin{bmatrix} I_1 \\ \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = L^{-1}\begin{bmatrix} \int_{0^-}^{t} v_1(\tau)\,d\tau \\ \int_{0}^{t} v_2(\tau)\,d\tau \end{bmatrix} + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Special Case: $L^{-1} = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}$



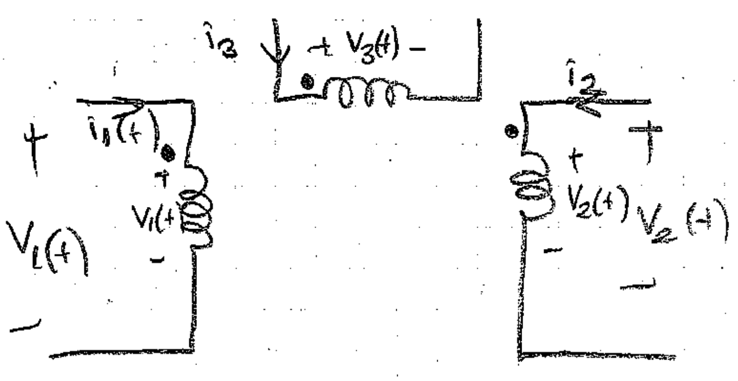
Initial Cond. for Mutual Ind.

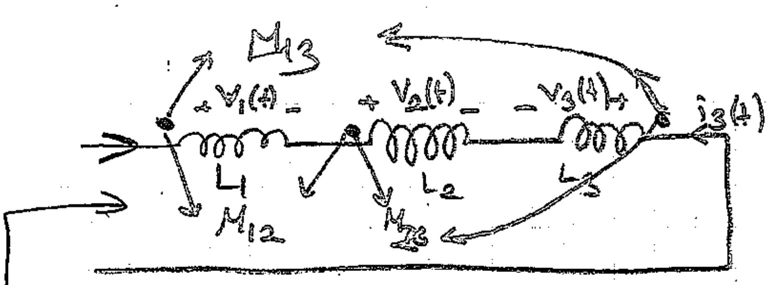


Coupling b/w 1st & 2nd winding

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} L_1 & M_{12} & M_{13} \\ M_{21} & L_2 & M_{23} \\ M_{31} & M_{32} & L_3 \end{bmatrix}}_{L} \begin{bmatrix} \frac{di_1(t)}{dt} \\ \frac{di_2(t)}{dt} \\ \frac{di_3(t)}{dt} \end{bmatrix}$$

$$M_{kl} = M_{lk} \longrightarrow \underline{\underline{L}} = \underline{\underline{L}}^T \text{ (symmetric)}$$





$L_{eq} = ?$

What is the relation bw $i(t)$ and $v(t)$ of this component? (math. relation)

$$V(t) = V_1(t) + V_2(t) - V_3(t)$$

$$= [1 + 1 - 1] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_1(t) \\ \frac{d}{dt} i_2(t) \\ \frac{d}{dt} i_3(t) \end{bmatrix}$$

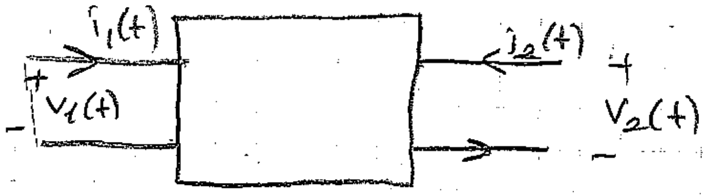
$$= \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i(t) \\ \frac{di(t)}{dt} \\ -\frac{d\,i(t)}{dt} \end{bmatrix}$$

$$\begin{bmatrix} L_1 + M_{21} & L_2 + M_{12} & -L_3 + M_{13} \\ -M_{31} & -M_{32} & M_{23} \end{bmatrix} D \begin{bmatrix} v(t) \\ i(t) \\ -i(t) \end{bmatrix}$$

$$V(t) = \left[(L_1 + M_{21} - M_{31}) + (L_2 + M_{12} - M_{32}) + (-L_3 + M_{13} + M_{23})\right] \frac{d}{dt}$$

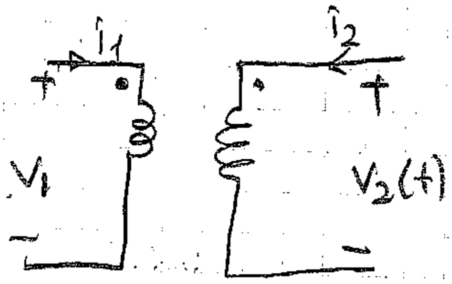
Power

Power & Energy Relations



$$P(t) = v_1(t)\, i_1(t) + V_2(t)\, i_2(t)$$

$$= \begin{bmatrix} v_1(t) \mid v_2(t) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$



$$P_{mut\ ind}(t) = \begin{bmatrix} v_1(t) \mid V_2(t) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$= \begin{bmatrix} D i_1 \; ; \; D i_2 \end{bmatrix} \underset{L}{L^T} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$P_{mut.\ Ind} = \begin{bmatrix} \frac{d}{dt} i_1(t) & \frac{d}{dt} i_2(t) \end{bmatrix} \underset{\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}}{L} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$= L_1\, i_1(t) \frac{d}{dt} i_1(t) + L_2\, i_2(t) \frac{d}{dt} i_2(t) + M \underbrace{\left[i_1(t) \frac{d}{dt} i_2(t) + i_2(t) \frac{d i_1}{dt} \right]}_{\frac{d}{dt}\left(i_1(t)\, i_2(t)\right)}$$

Watts.

Energy: $\underset{\text{Joule}}{E} = \int_{-\infty}^{t} \underset{\frac{\text{Joule}}{\text{sec}} = \text{Watts}}{P(\tau)}\, d\tau$

$$\boxed{P(t) = \frac{dW(t)}{dt}}$$

$$w(t) = \int_{-\infty}^{t} \left[\frac{d}{dt} i_1(t) \;\; \frac{d}{dt} i_2(t) \right] \underline{\underline{L}} \begin{bmatrix} i_1(t) \\ i_2 \end{bmatrix}$$

$$= \frac{1}{2} \left[i_1(t) \;\; i_2(t) \right] \underline{\underline{L}} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} \Bigg|_{t'=-\infty}^{t'=t}$$

Assuming $i_1(t) = i_2(t) \Big|_{t=-\infty} = 0$

$$\boxed{W(t) = \frac{1}{2} \left[i_1(t) \;\; i_2(t) \right] \underline{\underline{L}} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}}$$

$$= \frac{1}{2} \underline{i}^T \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \underline{i}$$

$$\boxed{W(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)}$$ Joule

If component is a passive component, $W(t) \geq 0 \;\; \forall t$ should be satisfied. Then, for a mutual inductor

$$W(t) \geq 0 \rightarrow \frac{1}{2} i^T \underline{\underline{L}} i \geq 0 \quad \forall t \; (\forall \underline{i} \text{ vectors})$$

$\underline{\underline{L}}$: matrices satisfying i_L^T satisfying $i_L^T \underline{\underline{L}} i \geq 0$

$\underline{i}^T \underline{\underline{L}} \underline{i} \geq 0 \rightarrow \forall i$ is called positive semi-definite matrice

and $L_{2\times2}$ is positive-semi definite

if $L_1 \geq 0$
$L_2 \geq 0$
$\det(\underline{\underline{L}}) \geq 0$

$\underline{\underline{L}}$: Mutual Inductances Matrix

$$\underline{\underline{L}} \rightarrow L_1 \geq 0, \quad L_2 \geq 0, \quad \det(\underline{\underline{L}}) \geq 0 \rightarrow L_1L_2 - M^2 \geq 0$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \quad L_1L_2 - M^2 \geq 0$$

If $L_1 L_2 - M^2 = 0 \xrightarrow{\text{Passive Mut. Ind.}} \det(\underline{\underline{L}}) = 0 \rightarrow L_1 L_2 - M^2 \geq 0$

$$\frac{M^2}{L_1L_2} \leq 1$$

Coupling coefficient of mutual inductor $\leftarrow k = \dfrac{M}{\sqrt{L_1L_2}} \rightarrow |k| \leq 1$

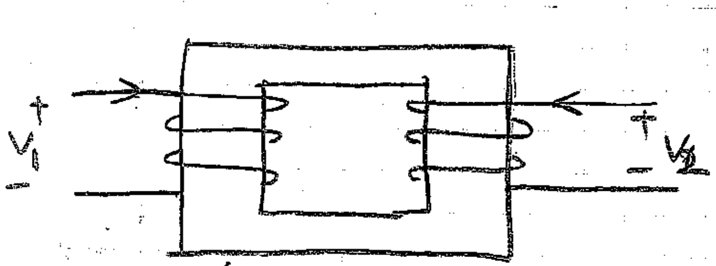
|Coupling coeff.| ≤ 1

Ideal Transformers

Mutual Inductor with $k = 1$ (no ohmic losses)

↓

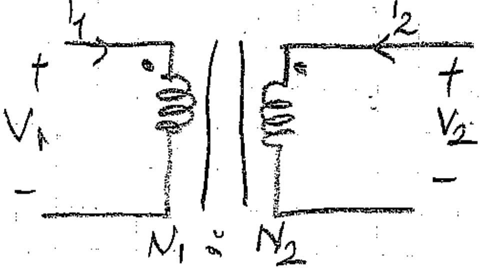
Ideal Transformers



$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{Voltage directly proportional}$$

$$\frac{i_1}{i_2} = -\frac{N_2}{N_1} \quad \text{Current inversely proportional}$$

Circuit Symbol



N_1 : Turn ratio of primary side

N_2 : " secondary "

$$p(t) = \begin{bmatrix} v_1(t) & v_2(t) \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

$$= v_1(t)\, i_1(t) + v_2(t)\, i_2(t)$$

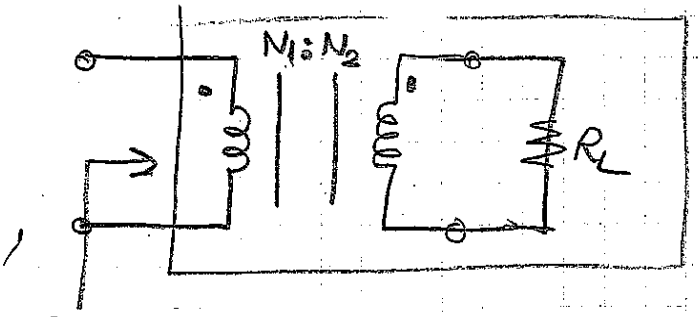
$$v_1(t)\, i_1(t) + \left(\frac{N_2}{N_1} v_1(t)\right)\left(-\frac{N_1}{N_2} i_1(t)\right)$$

$$= 0 \qquad \forall t$$

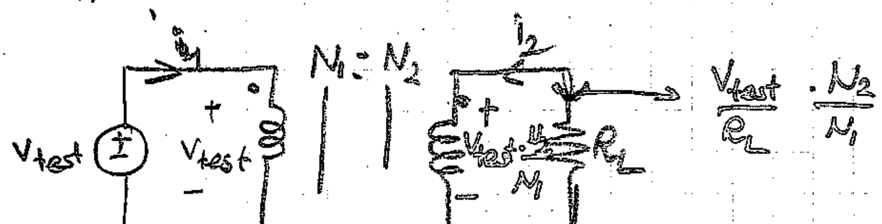
** Ideal transform does NOT absorb / deliver any power at any time

!! Important Property :

Resistance Reflection (Impedance)



$R_{th} = ?$



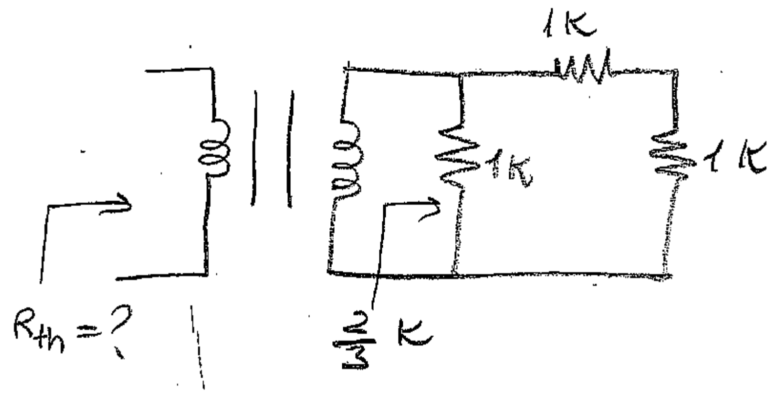
$$R_{Th} = \frac{V_{test}}{I_{test}}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{V_{test}}{i_1} = \frac{V_{test}}{\frac{V_{test}}{R_L} \cdot \left(\frac{N_2}{N_1}\right)^2} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

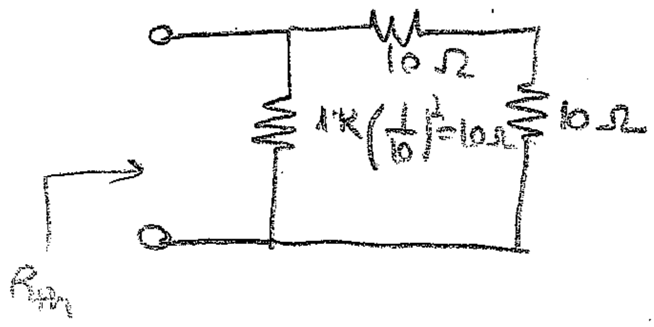
$i_1 = -\frac{N_2}{N_1} i_2$

R_{Th}: resistance seen from primary side

R_L: resistance seen from secondary side



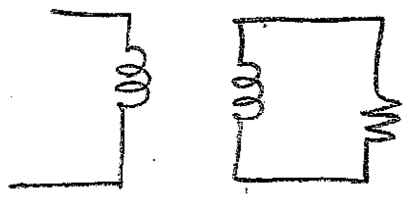
$$R_{Th} = \frac{2}{3}K \cdot \left(\frac{1}{10}\right)^2 = \frac{20}{3}\,\Omega$$

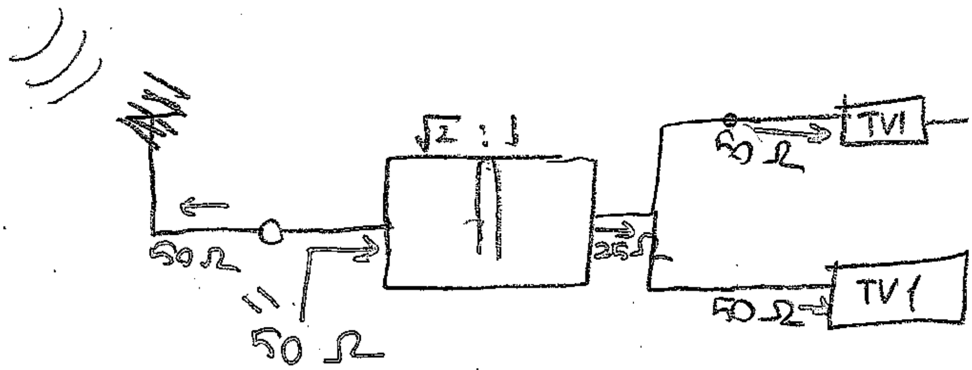


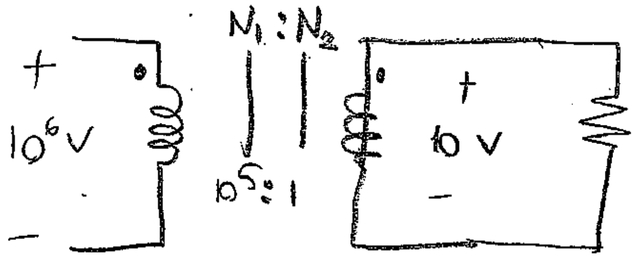
Comment: It is possible to reflect all resistors to the other side

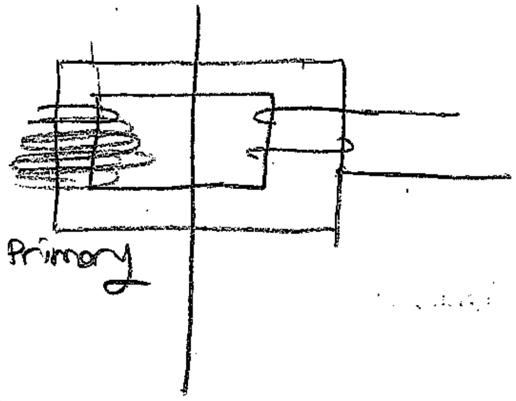
Use of Transformers

1. Impedance / Resistance Matching









Two sides are electrically isolated
(magnetically connected)

3.) Step-Up / Step-Down

Step-Up Voltage increases

Step-Down: " decreases