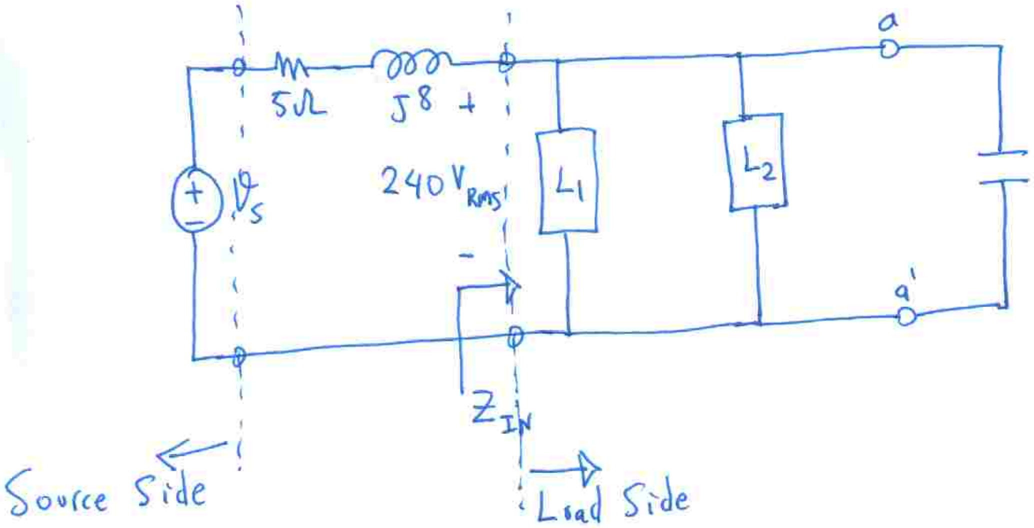
EE 202

A.C. Power Analysis Example

<u>Problem</u>: In the following circuit, $V_L = 240\ V_{RMS}$ at all times.
The loads operate under following conditions

L_1: absorbs 180 Watts and 240 Vars
L_2: absorbs 600 VA at 0.6 p.f. lagging



Operating frequency = 50 Hz

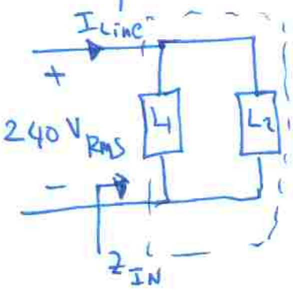
a) Assume there is no capacitor between a-a' lines. Find the input impedance at load side, source voltage in RMS, p.f. on the source side and also calculate the efficiency of the system. (Efficiency is the ratio of real power delivered to the load side and real power generated by the source.

b) Now, a capacitor is connected between a-a' terminals. Find C in Farad's such that efficiency is maximized. Find V_s in RMS.

c) Find C such that load side p.f. is 0.9 lagging. Find V_s after the connection of the Capacitor.

Solution:

a) No capacitor



$$S_{L_1} = 180 + j\,240$$

$$S_{L_2} = 600 \angle \cos^{-1}(0.6) = 360 + j\,480$$

$$+ \;\overline{\quad S_{Total}^{Load} = 540 + j\,720 \quad}$$

I

$$\left| S_{Total}^{Load} \right| = \underbrace{V_{Load}^{RMS}}_{240\,V_{RMS}} \; \underbrace{I_{Load}^{RMS}}_{I_{Line}^{RMS}} \implies \left| 540 + j720 \right| = 240 \cdot I_{Line}^{RMS}$$

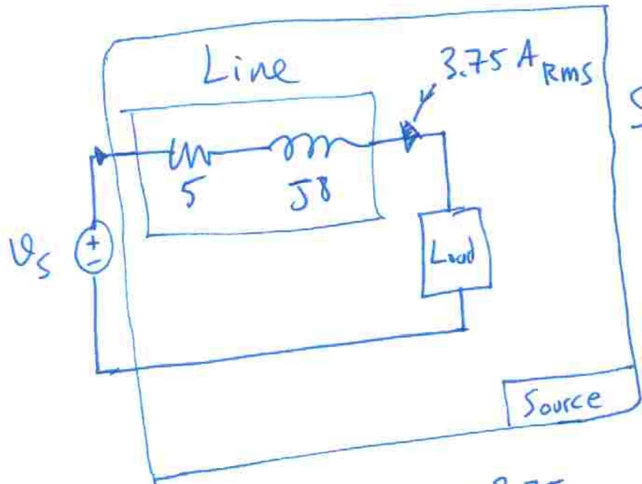
$$I_{Line}^{RMS} = \frac{900}{240} = 3.75\ A_{RMS}$$

II

$$S_{Total}^{Load} = \left(I_{Line}^{RMS} \right)^2 Z_{IN} \longrightarrow Z_{IN} = \frac{540 + j720}{(3.75)^2} = 38.4 + j\,51.2$$

$$= 64 \angle 53.1^\circ$$

III



$$S_{Line} = \left(I_{Line}^{RMS} \right)^2 \cdot (5 + j8)$$

$$= 70.3125 + j\,112.5$$

$$S_{Source} = S_{Line} + S_{Total}^{Load}$$

$$= 610.3125 + j\,832.5$$

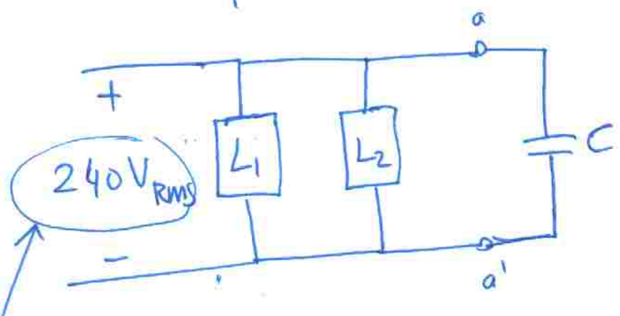
$$\left| S_{Source} \right| = v_S^{RMS} \cdot I_{Line}^{RMS} \longrightarrow v_S^{RMS} = \frac{|610.3125 + j832.5|}{3.75} = 275.26\ V_{RMS}$$

S_{Source}: 610.3125+j832.5; v_S^{RMS}: ?; I_{Line}^{RMS}: 3.75

$$\text{p.f on source side} = \cos\left(\tan^{-1} \frac{832.5}{610.3125} \right) = 0.59 \text{ lagging}$$

$$\text{Efficiency} = \frac{P_{delivered}\ (\text{Watts})}{P_{generated}\ (\text{Watts})} = \frac{540\ (\text{Watts})}{540 + 70.3125\ (\text{Watts})} = 88.48\ \%$$

b) With capacitor



Remains at 240 V_{RMS} as stated in the problem. (240 V_{RMS} is provided by the city network!)

$$\text{Efficiency} = \frac{P^{Load}_{delivered}}{P^{Load.}_{delivered} + P_{Line}}$$

Since both loads operate as in part a; that is

$$S^{Load\ w/Cap}_{Total} = 540 + j720$$

$$\text{Efficiency} = \frac{540}{540 + P_{Line}}$$

To maximize efficiency, we need to minimize line losses.

$$P_{Line} = \left(I^{RMS}_{Line}\right)^2 \underbrace{R_{Line}}_{5\Omega} \rightarrow$$ So we need to minimize I^{RMS}_{Line}

So we need to change the load side p.f. to unity!!

Before Compensation

$$S^{Before}_{Loads} = 540 + j720$$

$$I^{Before}_{Line} = 3.75\ A_{RMS}$$

After Compensation (Desired)

$$S^{After}_{Loads+Cap} = 540 + \cancel{j720} - \cancel{j720}$$
$$= 540 \quad \left(\text{unity p.f. is achieved}\right)$$

$$I^{after}_{Line} = \frac{540}{240} = 2.25\ A$$

$$S^{after}_{Line} = (2.25)^2(5 + j8) = 25.31 + j40.5$$

$$S^{after}_{Source} = 540 + 25.31 + j40.5$$

$$S_{Source}^{after} = 565.31 + j\,40.5$$

$$\left|S_{Source}^{after}\right| = 566.75 = V_{S_{RMS}}^{after} \cdot \underbrace{I_{Line}^{RMS}}_{2.25}$$

$$V_{S_{RMS}}^{after} = 251.89 \; V_{RMS}$$

$$\text{Efficiency} = \frac{540}{565.31} = 95.5\%$$

$$S_{compensator} = -j\,720 = \frac{\left(V_{cap}^{RMS}\right)^2}{X_c^*} \longrightarrow X_c^* = \frac{(240)^2}{-j\,720} = j\,80$$

$$X_c = -j\,80$$

$$X_c = \frac{1}{j\omega C} = \frac{1}{j(2\pi 50)C}$$

$$C = \frac{1}{(2\pi 50)\,80} = 39.8\,\mu F. \longleftarrow$$

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c) Before

$$S_{Loads} = 540 + j\,720$$

$$S_{cap} = -j\,459 \text{ VARS}$$

$$459 = \frac{(240)^2}{X_c^*} \rightarrow X_c = -j\,125$$

$$C = \frac{1}{(2\pi 50)125} = 25.3\,\mu F$$

After

$$S_{Loads + Cap} = 540 + j\,720 + S_{cap}$$

$$S_{cap} = -j\,Q_{cap}$$

$$= 540 + j\,(720 - Q_{cap}).$$

$$\text{p.f. (desired)} = 0.9$$

$$540 \tan(25.84) = 261$$

$$\cos^{-1}(0.9) = 25.84^\circ$$

$$\text{Then } 720 - Q_{cap} = 261 \longrightarrow \boxed{Q_{cap} = 459}$$