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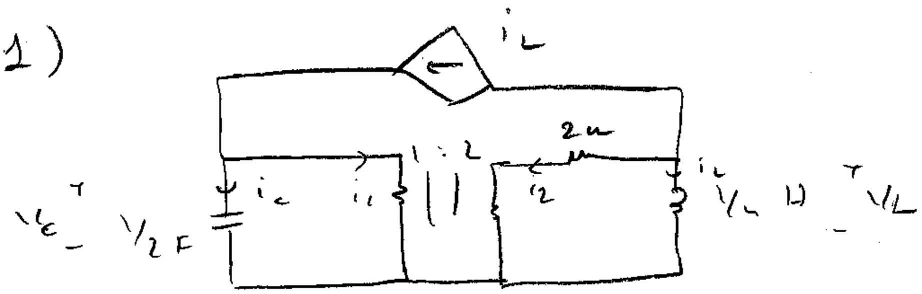
Section 3

EE-202

Homework -I-

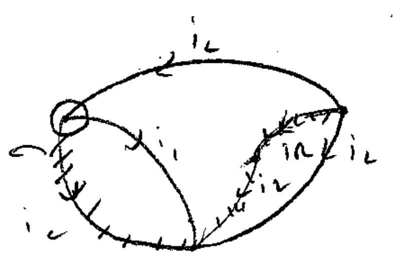
1)



By using state equations:

Graph:

Tree



State Variables: $\{V_c(t), i_L(t)\}$

$\frac{i_1}{i_2} = \frac{N_2}{N_1}$ $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

a) Fundamental Loop for $i_L(t)$ ($V_c(t)$)

$V_R + V_2 - V_L = 0$ $V_L = L\frac{di_L(t)}{dt}$

$V_2 = \frac{V_1 N_2}{N_1} = \frac{V_c N_2}{N_1}$

$V_R = i_R \cdot R$

$\hookrightarrow i_R \overset{FCS}{=} -2i_L$

Hence

$$-2i_L \cdot R + \frac{V_c N_2}{N_1} - L\, i_L'(t) = 0$$

$$i_L'(t) + 16\, i_L - 8 V_c = 0$$

Fundamental cut-set for $V_c(t)$, $(i_L(t))$

$i_c + i_1 - i_L = 0$

$\hookrightarrow \quad i_1 = -\dfrac{N_2}{N_1} i_2 = -\dfrac{N_2}{N_1} i_R = \dfrac{2N_2}{N_1} i_L$

Hence. $\qquad\qquad\qquad\qquad = \underline{4 i_L}$

$v_c^{\circ}(t) + 6 i_L(t) = 0$

in matrix form

$$\begin{bmatrix} V_c^{\circ}(t) \\ i_L^{\circ}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -6 \\ 8 & -16 \end{bmatrix}}_{\underline{L}} \begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix}$$

For a single excitation, let

$$\begin{bmatrix} V_c(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{\lambda t}$$

$$\lambda \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \cancel{e^{\lambda(t)}} = \underline{L} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \cancel{e^{\lambda t}}$$

$$\Rightarrow \underbrace{[\lambda I - \underline{L}]}_{\text{not invertible}} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

not invertible $\Rightarrow$ otherwise $\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$ trivial solution only!

Hence $\det[\lambda I - \underline{L}] = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & -6 \\ 8 & -16-\lambda \end{vmatrix} = 0 \Rightarrow \lambda(16+\lambda) + 24 = 0$$

$\lambda^2 + 16\lambda + 48 = 0$

$\lambda_1 = -12$

$\lambda_2 = -4$

For $\lambda_1 = -12$

$$\begin{bmatrix} 12 & -6 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2\alpha_1 = \alpha_2 \Rightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

For $\lambda_2 = -4$

$$\begin{bmatrix} 4 & -6 \\ 8 & -12 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2\gamma_1 = 3\gamma_2 \Rightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Let initial conditions $\Rightarrow$ $V_c(0) = V_0$

$I_L(0) = I_0$

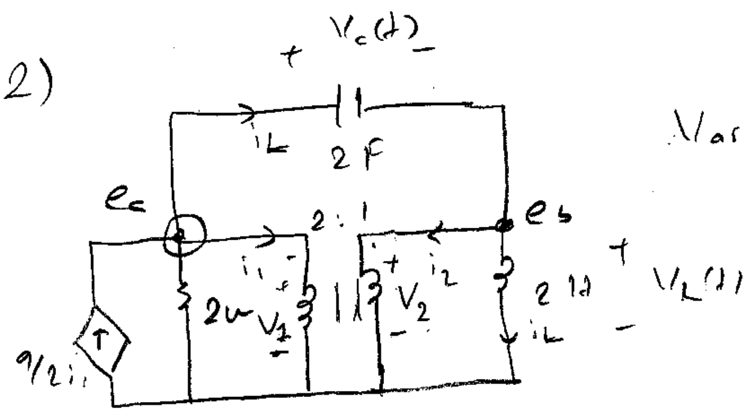
Hence for the first excitation;

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{\lambda \cdot 0} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \underline{\underline{2V_0 = I_0}}$$

For second excitation;

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{\lambda \cdot 0} \Rightarrow \underline{\underline{2V_0 = 3I_0}}$$

2)



Variables: e_a, e_b, i_1, i_2, i_L

Equations

1) $\dfrac{i_1}{i_2} = -\dfrac{N_2}{N_1}$ 2) $\dfrac{V_1}{V_L} = \dfrac{N_1}{N_2} = \dfrac{e_a}{e_b}$

3) $2\dfrac{di_L(t)}{dt} = V_L(t) = e_b$

4) $\frac{-9}{2} i_L + \frac{e_a}{2} + i_1 + \frac{2\,d(e_a - e_b)}{dt} = 0$

5) $2\frac{d}{dt}(e_b - e_a) + i_L + i_L = 0$

Matrix Form

$$\begin{bmatrix} 1/2 + 2D & -2D & -9/2 & 1 & 0 \\ -2D & 2D & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & -1 & 2D & 0 & 0 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ i_L \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For only one mode excitation;

$$\begin{bmatrix} e_a \\ e_b \\ i_L \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} e^{\lambda t}$$ → solve for λ's; hence

$\underline{\underline{L}}$ matrix

$$\begin{bmatrix} 1/2 + 2\lambda & -2\lambda & -9/2 & 1 & 0 \\ -2\lambda & 2\lambda & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & -1 & 2D & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} \cancel{e^{\lambda t}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For non-trivial solution; $\det[\underline{\underline{L}}] = 0$.

$\det[\underline{\underline{L}}] = 4\lambda^2 + 4\lambda - 8$

$\lambda = -2, 1$ ⇒ natural frequencies

To have only bounded currents and voltages; only excite $\lambda = -2$; hence put $\lambda = -2$

$$\begin{bmatrix} -7/2 & 4 & -9/2 & 1 & 0 \\ 4 & -4 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 1 & -2 & 0 & 0 & 0 \\ 0 & -1 & -4 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} e^{-2t} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\begin{bmatrix} e_a \\ e_b \\ i_L \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} e^{-2t}$

By Gaussian Elimination;

$$\begin{bmatrix} 1 & 0 & 8 & 0 & 0 \\ 0 & -1 & -4 & 0 & 0 \\ 0 & 0 & -15 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow 1 \text{ free variable.}$$

Set α_3 : free

$\alpha_1 = -8\alpha_3$ $\qquad \alpha_5 = 15\alpha_3$

$\alpha_2 = -4\alpha_3$ $\qquad \alpha_4 = \dfrac{-\alpha_5}{2} = \dfrac{-15\alpha_3}{2}$

Hence;

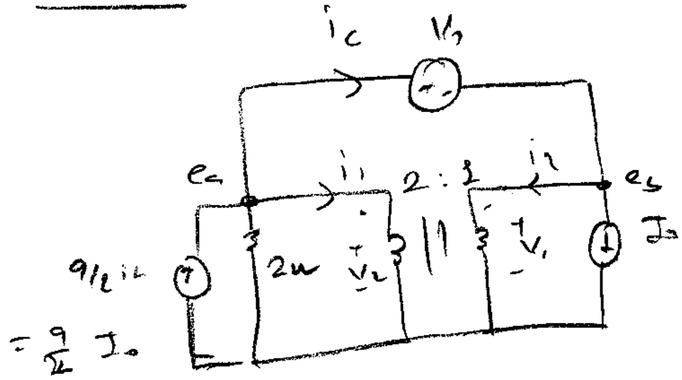
$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} \alpha \begin{bmatrix} -8 \\ -4 \\ 1 \\ -15/2 \\ 15 \end{bmatrix} //$$

use initial conditions;

$V_c(0) = V_0 \qquad i_L(0) = I_0$

then find $e_a(0)$, $e_b(0)$, $i_c(0)$, $i_1(0)$, $i_2(0)$,

at $t = 0^+$



since $V_2 = e_a$

$V_1 = e_b$

and $\frac{V_2}{V_1} = \frac{2}{1}$

$e_a = 2e_b$

KVL

$-e_a + V_0 + e_b = 0 \qquad (e_a = 2e_b)$

$e_b = V_0 \qquad e_a = 2V_0 \qquad$ at $t = 0$

$\frac{i_1}{i_2} = -\frac{N_2}{N_1} \Rightarrow \qquad i_2 = -2i_1$

KCL at e_a

$-\frac{9}{2} I_0 + \frac{2V_0}{2w} + i_1 + i_c = 0$

KCL at e_b

$i_2 + I_0 - i_c = 0$

$\Rightarrow \quad i_1 = \frac{-7}{2} I_0 + V_0$

$i_2 = 7I_0 - 2V_0$

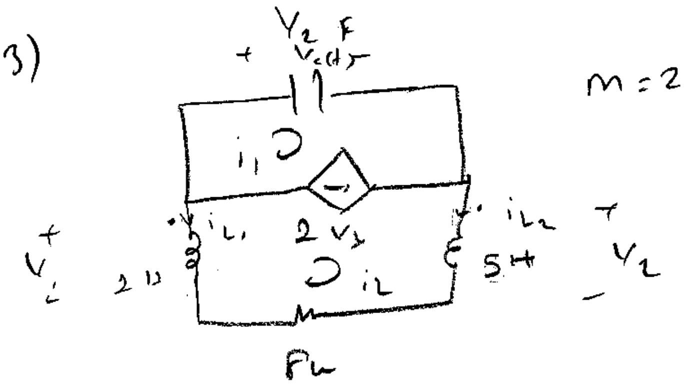
Put them into equation; at $t=0$

$$\begin{bmatrix} 2V_0 \\ V_0 \\ I_0 \\ -7/2\, I + V_0 \\ 7I - 2V_0 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} \alpha \begin{bmatrix} -8 \\ -4 \\ 1 \\ -11/2 \\ 11 \end{bmatrix}$$

By taking this proportionality into considerations for only 1 excitation for $\lambda = -2$

$$\begin{bmatrix} V_o \\ I_o \end{bmatrix} = \alpha \begin{bmatrix} -4 \\ 1 \end{bmatrix} \quad ///$$

3)



M = 2 H

$2V_x + i_2 = i_1$

$V_x = V_1$

<u>Mesh equation</u> (supermesh)

$$-V_1 + V_{c_1} + V_2 + 8 i_2 = 0$$

$$V_{c_1} = V_o + 2\int_0^t i_1(t)\, dt$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} i'_{L_1} \\ i'_{L_2} \end{bmatrix} \qquad \underline{-i_{L_1} = i_{L_2} = i_2}$$

$$V_1 = i'_2 \qquad V_2 = 3 i'_2$$

<u>Equation</u>

$$V_o + 2\int_0^t i_1(t)\, dt - i'_2 + 3i'_2 + 8 i_2 = 0 \quad (*)$$

<u>Constraint</u>

$$2 i'_2 + i_2 - i_1 = 0$$

To eliminate integral, let operate "D" on (*)

$$\Rightarrow 2 i_1(t) + 2 i''_2 + 8 i'_2 = 0$$

Matrix equation

$$\begin{bmatrix} -1 & 2D+1 \\ 2 & 2D^2+8D \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} e^{\lambda t}$ (single mode excitation)

$$\underbrace{\begin{bmatrix} -1 & 2\lambda+1 \\ 2 & 2\lambda^2+8\lambda \end{bmatrix}}_{\underline{M}} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} e^{\lambda t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

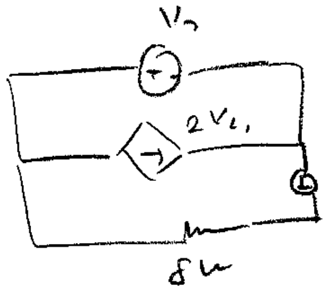
$$\underline{M} \Rightarrow \det[\underline{M}] = 0$$

$$(2\lambda^2+8\lambda) + 2(2\lambda+1) = 0$$

$$\lambda^2+6\lambda+1 = 0 \Rightarrow \lambda = 3 \pm 2\sqrt{2} \rightarrow \text{natural frequencies}$$

Hence $i_2 = \alpha_1 e^{(3+2\sqrt{2})t} + \alpha_2 e^{(3-2\sqrt{2})t}$

at $t=0^+$, assume an equivalent inductor $L_{eq} = 2H$, by the matrix representing inductors



$I_{Leq} = I_0$

$V_c(0) = V_0$

$V_{L eq}(0) = -V_0 + 8I_0$

$2 \cdot I'_{Leq} = V_{Leq}$

$$I'_{Leq} = \frac{-V_0 + 8I_0}{2}$$

Conditions at $t=0$

$\alpha_1 + \alpha_2 = V_0$

$(3+2\sqrt{2})\alpha_1 + (3-2\sqrt{2})\alpha_2 = \dfrac{-V_0+8I_0}{2} \Rightarrow$

$$\alpha_1 = \left(\frac{2\sqrt{2} - 5/2}{4\sqrt{2}}\right) V_0 + \frac{1}{\sqrt{2}} I_0 \qquad \alpha_2 = \frac{(5/2 + 2\sqrt{2})}{4\sqrt{2}} V_0 - \frac{1}{\sqrt{2}} I$$

since $-L\dfrac{dI_{Leq}}{dt} = -2 \cdot \dfrac{dI_{Leq}}{dt} = V_x(t) = V_L(t)$

$$V_x(t) = -\alpha_1(3+2\sqrt{2})e^{(3+2\sqrt{2})t} - \alpha_2(3-2\sqrt{2})e^{(3-2\sqrt{2})t}$$

6) a) homogeneous solution;

let $x^h(t) = Ae^{\lambda t}$ $\Rightarrow$ char. eqn: $\lambda^3 + \lambda^2 + 2\lambda + 2 = 0$

$\lambda = -1, -\sqrt{2}j, \sqrt{2}j$

Hence $x^h(t) = \alpha_1 e^{-t} + \alpha_2 \sin(\sqrt{2}t) + \alpha_3 \cos(\sqrt{2}t)$

b) (i) $3e^{2t} = u_s(t)$

$(D^3 + D^2 + 2D + 2)\, x(t) = 3(D+2)\, u_s(t)$

$= 3(6e^{2t} + 6e^{2t})$

$= 36e^{2t}$

For $x^p(t) \rightarrow$ guess $= Ae^{2t} \rightarrow$ put it into D.E.

$(8A + 4A + 4A + 2A)e^{2t} = 36e^{2t} \Rightarrow A = 2$

Hence $x^p(t) = 3e^{2t}$

(ii) $4e^{-t} = u_s(t)$

$(D^3 + D^2 + 2D + 2)\, x(t) = 3(D+2)4e^{-t}$

$= 12(-e^{-t} + 2e^{-t})$

guess $\Rightarrow x^p(t) = Ae^{-t}.t$ $\qquad = 12e^{-t}$

$x'^p(t) = Ae^{-t} - Ate^{-t}$

$x''^p(t) = -2Ae^{-t} + Ate^{-t}$

$x'''^p(t) = 3Ae^{-t} - Ate^{-t}$

Put them into equation; then

$3Ae^{-t} = 12e^{-t} \Rightarrow A = 4$

iv) $u_s(t) = 5\cos(2t+30°)$

let $x_p(t) = A\cos(2t+30°) + B\sin(2t+30°)$

$x'_p(t) = -2A\sin(2t+30°) + 2B\cos(2t+30°)$

$x''_p(t) = -4A\cos(2t+30°) - 4B\sin(2t+30°)$

$x'''_p(t) = 8A\sin(2t+30°) - 8B\cos(2t+30°)$

since $(D^3+D^2+2D+2)x_p(t) = (3D+6)\,5\cos(2t+30°)$

put the above into equation

$(4A-2B)\sin(2t+30°) + (-4B-2A)\cos(2t+30°) = -30\left[\sin(2t+30°) + \cos(2t+30°)\right]$

Hence;

$$\left.\begin{aligned} -30 &= 4A-2B \\ 30 &= -4B-2A \end{aligned}\right] \Rightarrow A=-3 \quad B=-1$$

Hence $x_p(t) = -3\cos(2t+30°) - \sin(2t+30°)$

v) $u_s(t) = 5e^{-t}\cos(t+30°)$

let $x_p(t) = Ae^{-t}\cos(t+30°) + Be^{-t}\sin(t+30°)$

$x'_p(t) = (-A-B)e^{-t}\sin(t+30°) + (B-A)e^{-t}\cos(t+30°)$

$x''_p(t) = 2Ae^{-t}\sin(t+30°) + (-2B)e^{-t}\cos(t+30°)$

$x'''_p(t) = (2B-2A)e^{-t}\sin(t+30°) + (2A+2B)e^{-t}\cos(t+30°)$

since $(D^3+D^2+2D+2)x_p(t) = (3D+6)\,5e^{-t}\cos(t+30°)$
$= 15e^{-t}(\cos(t+30°) - \sin(t+30°))$

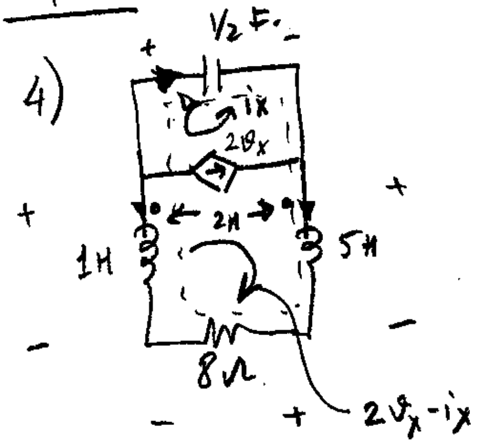
put the above into equation; then we get

$(2B+2A)e^{-t}\sin t + (2A+2B)e^{-t}\cos t = 15e^{-t}(\cos(t+30°) - \sin(t+30°))$

$$15 = 2A + 2B$$
$$-15 = 2B - 2A$$
$$+ \overline{}$$
$$A = 15/2 \qquad B = 0$$

Hence $\quad x^p(t) = 15/2 \; e^{-t} \cos(t + 30°)$ //

4)



Mesh Analysis:

2 mesh
1 current source } 1 mesh current as unknown.

KVL outer mesh:

$$V_c(t) + V_{5H}(t) + V_{8\Omega}(t) - V_{1H}(t) = 0$$

$$\begin{bmatrix} V_{5H}(t) \\ V_{1H}(t) \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{d}{dt} i_{5H}(t) \\ \frac{d}{dt} i_{1H}(t) \end{bmatrix}$$

$i_{5H}(t)$: $2v_x - i_x$

$i_{1H}(t)$: $-(2v_x - i_x)$

$$V_{5H}(t) - V_{1H}(t) = 2\, d/dt\,(2v_x - i_x)$$

$$\left(V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} -i_x(t')\,dt'\right) + 2\frac{d}{dt}(2v_x - i_x) + 8(2v_x - i_x) = 0$$

$$v_x = V_{1H} \longrightarrow \quad V_{1H} = [2 \quad 1]\begin{bmatrix} \frac{d}{dt}(2v_x - i_x) \\ \frac{d}{dt}\cdot -(2v_x - i_x) \end{bmatrix} = \frac{d}{dt}(2v_x - i_x)$$

V_{1H}: V_x

$$\boxed{v_x = \frac{d}{dt}(2v_x - i_x)} \longrightarrow \int_{0^-}^{t} v_x(t')\,dt' = 2v_x(t) - i_x(t) - [2v_x(0^-) - i_x(0^-)]$$

$$D^{-1} = \int_{0^-}^{t} (\ldots)\,dt'$$

$$v_c(0^-) + \frac{1}{C}\int_{0^-}^{t} -i_x(t')\,dt' + 2v_x + 8\left(D^{-1}\{v_x\} + 2v_x(0^-) - i_x(0^-)\right) = 0$$

$$v_c(0^-) - \frac{1}{C}D^{-1}\{i_x(t)\} + 2v_x + 8\left(D^{-1}v_x + 2v_x(0^-) - i_x(0^-)\right) = 0$$

$\downarrow D^2$

$$\rightarrow -\frac{1}{C}(D\,i_x) + 2D^2 v_x + 8(D\,v_x) = 0 \longrightarrow$$

$D\,i_x$: $(2D-1)v_x(t)$

C: $1/2$

$$\cancel{2}(1-2D)\,v_x(t) + \cancel{2}D^2 v_x(t) + \overset{4}{\cancel{8}}\,D\,v_x(t) = 0.$$

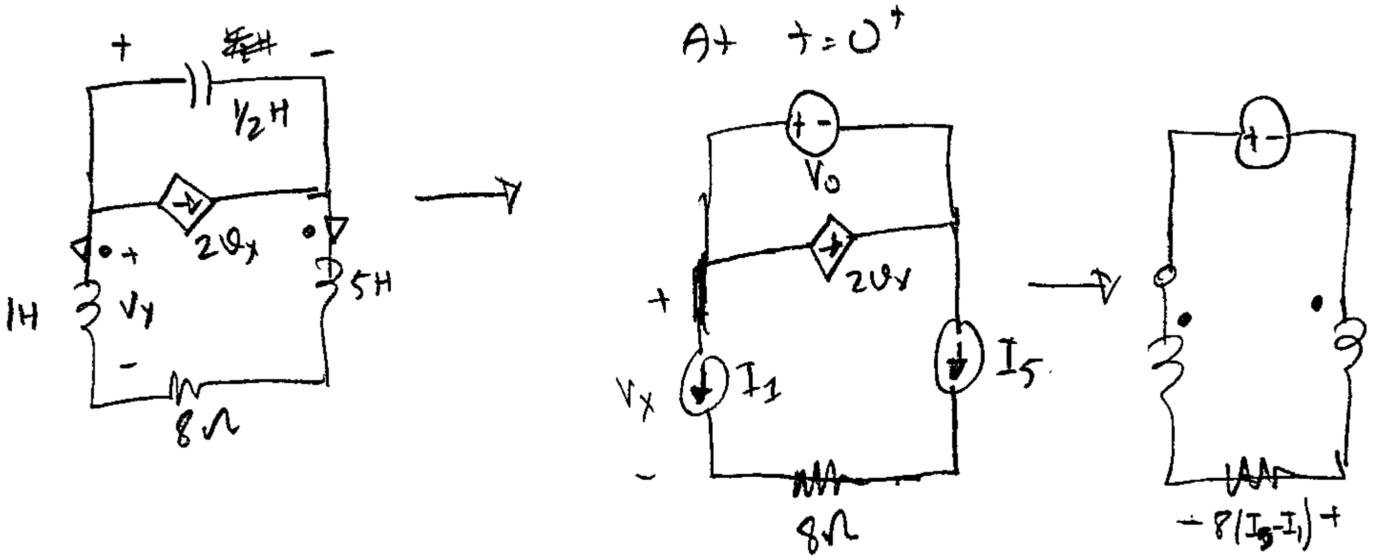
$$\left(D^2 + 2D + 1\right) v_x(t) = 0.$$

$$\lambda^2 + 2\lambda + 1 = 0 \rightarrow \lambda = \{-1, -1\}.$$

Then

$$v_x(t) = \alpha_1 e^{-t} + \alpha_2 t e^{-t}$$

To find α_1, α_2; we need $v_x(0^+)$ and $\dot{v}_x(0^+)$

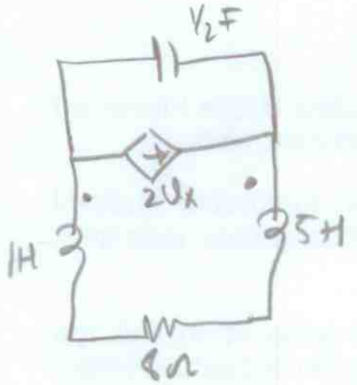


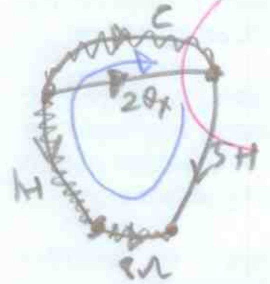
Finding $V_x(0^+)$ and $\dot{V}_x(0^+)$ is difficult with the methods that we have currently studied. (Voltage division across mutual inductors)

Assume $V_x(0^+) = A$, $\dot{V}_x(0^+) = B$ $\rightarrow$ Solution is $v_x(t) = \left(Ae^{-t} + (B+A)te^{-t}\right)$

ZPS I:

Pr 4 | State Eqn.





State var $= \{v_C, I_{5H}\}$

Fun-Cut Set:

$$C\dot{v}_C = -2v_x + I_{5H}$$

$$v_x = v_C + v_{5H} - V_{8\Omega}$$

$$\boxed{V_{8\Omega} = -8I_{5H}}$$

$$v_{5H} = 5\frac{d}{dt}i_{5H}(t) + 2\frac{d}{dt}i_{1H}(t)$$

$$= 5D(I_{5H}) + 2D(I_{1H}(t))$$

$-I_{5H}(t)$

$$\boxed{v_{5H} = 3D(I_{5H})}$$

$$\boxed{C\dot{v}_C = -16I_{5H} + 6D(I_{5H}) - 2v_C + I_{5H}} \quad \boxed{1}$$

Fun loop:

$-i_{5H}(t)$

$$\begin{bmatrix} V_{1H} \\ V_{5H} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} d/dt\, i_1(t) \\ d/dt\, i_5(t) \end{bmatrix}$$

$$V_{1H} = \frac{d}{dt}i_{5H}(t) \; ; \; V_{5H} = 3\frac{d}{dt}i_{5H}(t).$$

Fun loop Eqn: $V_{5H} = -V_C + V_{1H} + V_{8\Omega}$

$$3\frac{d}{dt}i_{5H}(t) = -V_C + \frac{d}{dt}i_{5H}(t) - 8i_{5H}(t)$$

$$\boxed{2\frac{d}{dt}i_{5H}(t) = -V_C - 8i_{5H}(t)} \quad \boxed{2}$$

Insert $\boxed{2}$ in $\boxed{1}$ $\rightarrow$ $\boxed{C\dot{v}_C = v_C + 9I_{5H}(t)} \quad \boxed{3}$

C: ½.

$$\begin{bmatrix} \dot{v}_c(t) \\ \dot{I}_{SH}(t) \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} 2 & 18 \\ \hline -1/2 & -4 \end{array}\right]}_{\underline{\underline{A}}} \begin{bmatrix} V_c \\ I_{SH} \end{bmatrix}$$

$$\det(\lambda \underline{\underline{I}} - \underline{\underline{A}}) \longrightarrow \det\left(\left[\begin{array}{c|c} \lambda-2 & -18 \\ \hline -1/2 & \lambda+4 \end{array}\right]\right) = (\lambda-2)(\lambda+4) - 9$$
$$= \lambda^2 + 2\lambda + 1.$$

$$\det(\lambda \underline{\underline{I}} - \underline{\underline{A}}) = 0 \longrightarrow \lambda = \{-1, -1\}.$$

Then.

$$\underline{x}(t) = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} e^{-t} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} t e^{-t}$$

$$\begin{bmatrix} v_c(t) \\ I_{SH}(t) \end{bmatrix} =$$

$$\begin{bmatrix} v_c(0^+) \\ I_{SH}(0^+) \end{bmatrix} = \begin{bmatrix} V_0 \\ I_{SH} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \longrightarrow$$

$\longrightarrow$ then $\underline{x}(t) = \begin{bmatrix} V_0 \\ I_{SH} \end{bmatrix} e^{-t} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} t e^{-t}$ $\leftarrow$ substitute in $\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t)$ $\longrightarrow$

to get $\longrightarrow$ $\overbrace{-\begin{bmatrix} V_0 \\ I_{SH} \end{bmatrix} e^{-t} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} (e^{-t} - te^{-t})}^{\dot{x}(t)} = \overbrace{\underline{\underline{A}} \begin{bmatrix} V_0 \\ I_{SH} \end{bmatrix} e^{-t} + \underline{\underline{A}} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} t e^{-t}}^{A x(t)}$

$$e^{-t}\Bigg(\underbrace{\begin{bmatrix} -V_0 \\ -I_{SH} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} - \underline{\underline{A}} \begin{bmatrix} V_0 \\ I_{SH} \end{bmatrix}}_{①}\Bigg) - te^{-t}\Bigg(\underbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + \underline{\underline{A}} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_{②}\Bigg) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

① should be $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ② should be $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

① $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = A \begin{bmatrix} V_0 \\ I_{5H} \end{bmatrix} + \begin{bmatrix} V_0 \\ I_{5H} \end{bmatrix} = \begin{bmatrix} 3V_0 + 18 I_{5H} \\ -\frac{1}{2} V_0 - 3 I_{5H} \end{bmatrix}$

② $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + \underline{\underline{A}} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \overset{?}{=} \underline{0} \longrightarrow \quad A \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} -3V_0 - 18 I_{5H} \\ \frac{1}{2} V_0 + 3 I_{5H} \end{bmatrix}$

$\downarrow$

then $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + A \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

Then

$$\begin{bmatrix} v_c(t) \\ I_{5H}(t) \end{bmatrix} = \begin{bmatrix} V_0 \\ I_{5H} \end{bmatrix} e^{-t} + \begin{bmatrix} 3V_0 + 18 I_{5H} \\ -\frac{1}{2} V_0 - 3 I_{5H} \end{bmatrix} t e^{-t}$$

where $V_c(0^-) = V_0$
$I_{5H}(0^-) = I_{5H}$