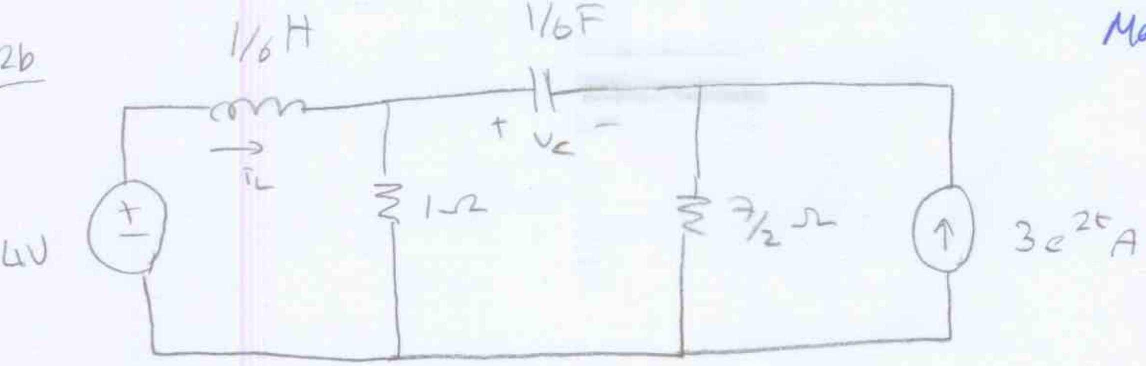
ZPS I

12b

Olgu Kıvanç ONACAK
Mart 2013



Particular Solution = ?

Since we have 2^{nd} order dynamic circuit the differential eqn. is in the form

$$(D^2 + \alpha D + \beta)\, x(t) = A e^{s_0 t} + B e^{s_1 t}$$

Branch Voltage or current

DC voltage source $s_0 = 0$

Current Source $s_1 = 2$

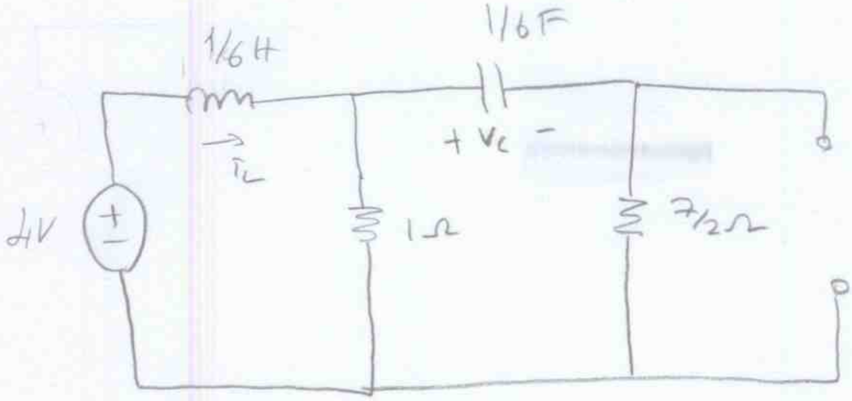
For every branch current / Voltage the particular solution becomes: $K_0 e^{s_0 t} + K_1 e^{s_1 t}$

If there is no current source $\Rightarrow K_0 e^{s_0 t}$, $s_0 = 0$

If there is no Voltage Source $\Rightarrow K_1 e^{s_1 t}$, $s_1 = 2$

Therefore it is possible to apply superposition principle to find the particular solution!

① Only Voltage Source

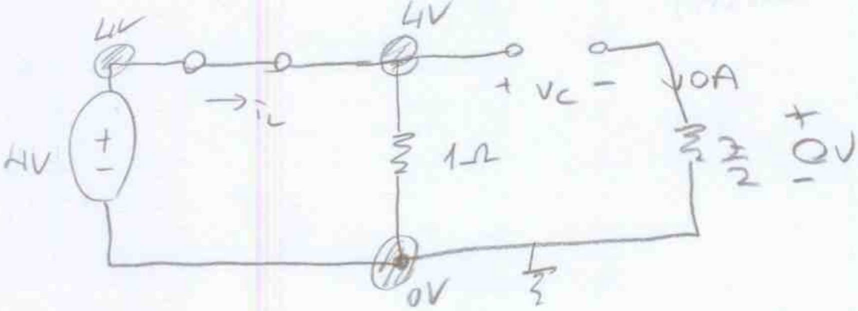


Our guess is: $V_C(t) = K_0 e^{s_0 t}$, $s_0 = 0$, $V_C(t) = K_0$

$I_L(t) = L_0 e^{s_0 t}$, $s_0 = 0$, $I_L(t) = L_0$

$i_C^{P_I} = C \cdot \dot{V_C}(t) = 0$ Capacitor acts like an open circuit

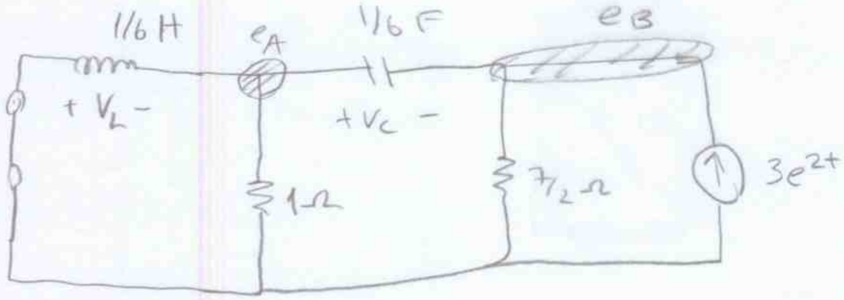
$V_L^{P_I} = L \cdot \dot{i_L}(t) = 0$, Inductor acts like a short circuit



$\frac{4-0}{1} = i_L^{P_I} = 4A$ $V_C^{P_I} = 4V$

$$\boxed{\begin{aligned} V_C^{P_I}(t) &= 4V \\ I_L^{P_I}(t) &= 4A \end{aligned}}$$

② Only Current Source



Guess: $V_C^{P_2} = K_1 e^{2t}$, $i_C^{P_2} = \frac{1}{3} K_1 e^{2t}$

$I_L^{P_2} = L_1 e^{2t}$, $V_L^{P_2} = \frac{1}{3} L_1 e^{2t}$

KCL at e_A:

$\frac{e_A}{1} - i_L + i_C = 0$ where: $-V_L = e_A$

$-\frac{1}{3} L_1 e^{2t} = e_A$

$-\frac{1}{3} L_1 \cancel{e^{2t}} - L_1 \cancel{e^{2t}} + \frac{1}{3} K_1 \cancel{e^{2t}} = 0$

* $K_1 = 4L_1$

KCL at e_B:

$-i_C - 3e^{2t} + \frac{e_B}{\frac{7}{2}} = 0$ where: $e_A - e_B = V_C^{P_2}(t)$

$-\frac{1}{3} L e^{2t} - e_B = K_1 e^{2t}$

$-\frac{K_1}{12} e^{2t} - e_B = K_1 e^{2t}$

$e_B = -\frac{13}{12} e^{2t}$

$-\frac{1}{3} K_1 \cancel{e^{2t}} - 3\cancel{e^{2t}} - \frac{13}{42} \cancel{e^{2t}} \cdot K_1 = 0$

$\frac{27}{42} K_1 = -3$

$\boxed{K_1 = -\frac{14}{3}}$

$\boxed{L_1 = -\frac{7}{6}}$

$$\boxed{\begin{aligned} V_C^{P_2}(t) &= -\frac{14}{3} e^{2t} \\ I_L^{P_2}(t) &= -\frac{7}{6} e^{2t} \end{aligned}}$$

Particular Solution becomes:

$V_C^P(t) = 4 - \frac{14}{3} e^{2t}$

$I_L^P(t) = 4 - \frac{7}{6} e^{2t}$