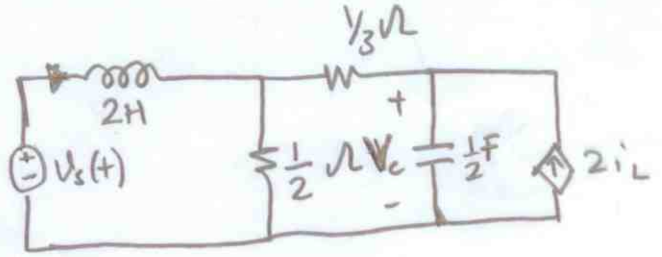


Find the particular solution for $V_c(t)$ given that $v_s(t) = Ke^{s_0 t}$.

where $s_0 \notin \{$Nat. frequencies of the circuit$\}$.

Soln:

$$\underbrace{v_k^{comp}(t)}_{k^{th}\text{ branch of the circuit}} = V_k^h(t) + v_k^P(t)$$

$$= \underbrace{\left(\alpha_{1k} e^{\lambda_1 t} + \alpha_{2k} e^{\lambda_2 t}\right)}_{\text{homogenous}} + \underbrace{\beta_k e^{s_0 t}}_{\text{particular.}} \quad \left(\text{since } \begin{array}{l} s_0 \neq \lambda_1 \\ s_0 \neq \lambda_2 \end{array}\right)$$

A KVL equation equation for the circuit can be written as

$$v_1^{comp}(t) + v_2^{comp}(t) + v_3^{comp}(t) = 0 \qquad (1)$$

It should be noted that Eq. (1) is valid for all "t". Hence

$$\underbrace{\left(v_1^h(t) + V_1^P(t)\right)}_{\text{this}} + \left(V_2^h(t) + V_2^P(t)\right) + \left(V_3^h(t) + V_3^P(t)\right) = 0$$

In every bracket (like "this"), we have exp. functions with exponents $\lambda_1 t$, $\lambda_2 t$ or $s_0 t$. We know that summation of three brackets is identical to zero for all t. Hence,

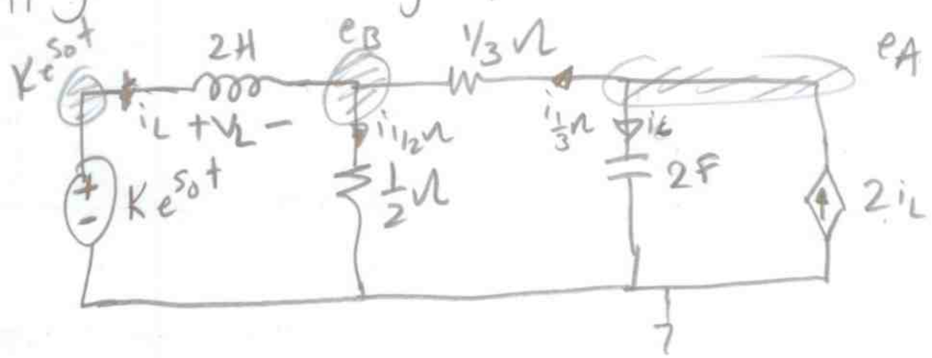
$$v_1^h(t) + v_2^h(t) + v_3^h(t) = 0, \quad \forall t$$

$$v_1^P(t) + v_2^P(t) + v_3^P(t) = 0, \quad \forall t$$

Since the homogenous solution contain only the exponentials with exponents $\lambda_1 t$ and $\lambda_2 t$, the summation of homogenous solutions should be equal to zero on its own! The particular soln. is no help to satisfy the equality, since it contains an independent exponential function. (2)

In this problem, we are only interested in particular solution; therefore we'll focus on $\underbrace{e^{s_0 t}}$ terms in the solution.

Let's apply node analysis,



<u>KCL at e_A :</u> $-2i_L(t) + i_C(t) + i_{\frac{1}{3}\Omega}(t) = 0 \quad , \quad \forall t$

of course, we refer to the complete solution, that is

$$-2\,i_L^{comp}(t) + i_C^{comp}(t) + i_{\frac{1}{3}\Omega}^{comp}(t) = 0 \quad , \quad \forall t$$

From earlier discussions, specific for the particular solution

$$-2i_L^{P}(t) + i_C^{P}(t) + i_{\frac{1}{3}\Omega}^{P}(t) = 0 \quad , \quad \forall t.$$

$$i^P_{\frac{1}{3}\Omega}(t) = \frac{e^P_A(t) - e^P_B(t)}{1/3}$$

$$i^P_c(t) = C\frac{dv^P_c(t)}{dt} = C\frac{d}{dt}\{e^P_A(t)\} = \overset{2F}{C}\,\dot{e}^P_A(t)$$

$$i^P_L(t) = ? \quad i^{comp}_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v^{comp}_L(z)\,dz$$

$$= i_L(0^-) + \frac{1}{L}\int_{0^-}^{t}[v_s(z) - e_B^{comp}(z)]\,dz.$$

$$= i_L(0^-) + \frac{1}{L}\int_{0}^{t}[v_s(z) - (e^P_B(z) + e^h_B(z))]\,dz.$$

Now, we know that particular soln. of the circuit for all circuit branches for both current and voltage variables is in the form $B_k e^{s_0 t}$.

an unknown scalar

Let $e^P_A(t) = Ae^{s_0 t}$, $e^P_B(t) = Be^{s_0 t}$ unknown to be found

Then

$$i^P_{\frac{1}{3}\Omega}(t) = 3(A - B)e^{s_0 t}$$

$$i^P_C(t) = 2As_0 e^{s_0 t}$$

$i^P_L(t)$: is the term of $i^{comp}_L(t)$ in the form $Be^{s_0 t}$.

$$\to i^{comp}_L(t) = i_L(0^-) + \frac{1}{\underset{2H}{L}}\int_{0^-}^{t}[\underbrace{(Ke^{s_0 z} - Be^{s_0 z})}_{(K-B)e^{s_0 z}} - e^h_B(z)]\,d$$

$$i_L^{comp}(t) = i_L(0^-) + \underbrace{\frac{1}{2}\frac{(K-B)}{s_0}e^{s_0 z}\Big|_{z=0}^{z=t}}_{\frac{1}{2}\frac{(K-B)}{s_0}\left[e^{s_0 t}\right] - \frac{1}{2}\frac{(K-B)}{s_0}} - \frac{1}{2}\int_0^t V_L(z)\,dz$$

In the complete solution for $i_L(t)$, only the term

$$\frac{1}{2}\frac{K-B}{s_0}e^{s_0 t}$$

is in the form required for particular solution; so

$$i_L^P(t) = \frac{1}{2}\frac{(K-B)}{s_0}e^{s_0 t}$$

KCL at e_A:

$$-2i_L^P(t) + i_C^P(t) + i_{\frac{1}{3}\Omega}^P(t) = 0 \leftarrow \text{only for particular soln.}$$

$$-\left(\frac{K-B}{s_0}\right)\cancel{e^{s_0 t}} + 2As_0\cancel{e^{s_0 t}} + 3(A-B)\cancel{e^{s_0 t}} = 0$$

$$\boxed{(2s_0+3)A + \left(\frac{1}{s_0}-3\right)B = \frac{K}{s_0}}$$

KCL at e_B:

$$i_{\frac{1}{2}}^P(t) - i_{\frac{1}{3}}^P(t) - i_L^P(t) = 0 \leftarrow \text{only for particular soln.}$$

$$2B\cancel{e^{s_0 t}} - 3(A-B)\cancel{e^{s_0 t}} - \frac{1}{2}\frac{(K-B)}{s_0}\cancel{e^{s_0 t}} = 0$$

$$\boxed{-3A + \left(2+3+\frac{1}{2s_0}\right)B = \frac{K}{2s_0}}$$

$$\begin{bmatrix} 2s_0+3 & \frac{1}{s_0}-3 \\ -3 & 5+\frac{1}{2s_0} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} \frac{K}{s_0}$$

$$\begin{bmatrix} 2s_0^2+3s_0 & 1-3s_0 \\ -6s_0 & 10s_0+1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 2s_0^2+3s_0 & 1-3s_0 \\ -6s_0 & 10s_0+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

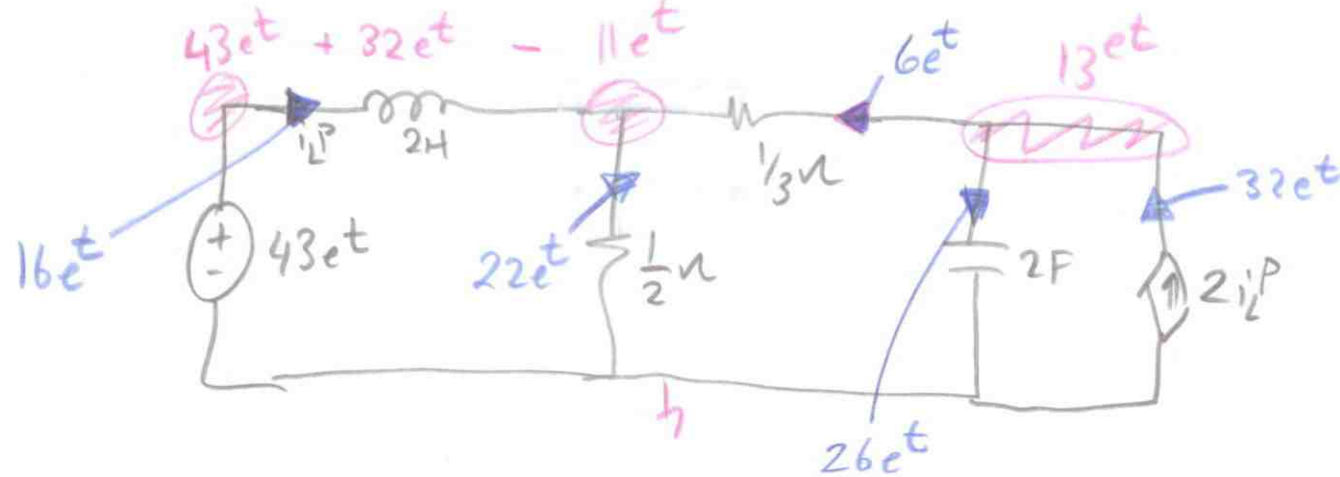
$$= \frac{1}{\Delta} \begin{bmatrix} 10s_0+1 & 3s_0-1 \\ 6s_0 & 2s_0^2+3s_0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} K.$$

$$\Delta = 20s_0^3+14s_0^2+9s_0$$

$$= \frac{1}{\Delta} \begin{bmatrix} 13s_0 \\ 2s_0^2+9s_0 \end{bmatrix} K = \begin{bmatrix} \frac{13}{20s_0^2+14s_0+9} \\ \frac{2s_0+9}{20s_0^2+14s_0+9.} \end{bmatrix} K.$$

Let $s_0 = 1 \rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 13/43 \\ 11/43 \end{bmatrix} K \rightarrow$ Let $K = 43 \rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 13 \\ 11 \end{bmatrix}$

Let's check the particular solution for $v_s(t) = 43e^t$



It can be checked that all KCL equations hold; hence the particular solution is correct!

Another check

Let $s_0 = -2 \rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 13/61 \\ 5/61 \end{bmatrix}$ Let $K = 61$

Then $v_s(t) = 61e^{-2t}$

$e_A^P(t) = 13e^{-2t}$, $e_B^P(t) = 5e^{-2t}$