Fall 2006

EE 503
Homework #1
Due: Sept. 29, 2006

Most of these problems require very few operations if you are comfortable with basic linear algebra methods. If you are having excessive difficulty, please consider a ***serious*** linear algebra refreshing; we will be using linear algebra in the graduate level DSP studies all the time. Strang's book is a standard and a good textbook for linear algebra. You can also watch Prof. Strang's teaching Linear Algebra on the web. http://ocw.mit.edu/OcwWeb/Mathematics/18-06Spring-2005/VideoLectures/index.htm (This is the first link coming up on a google search with the keyword "ocw linear algebra strang")

Submit only: Problems 22, 27, 28, 29 and 30. (The rest is for study.)

Reading Assignment: Section 2.3 from Hayes.

1. Show that inverse of a lower triangular is lower triangular.
2. Show that any matrix **A** can be expressed as **A=LU,** where **L** is a lower triangular matrix and **U** is upper triangular. (Hint: Gaussian elimination)
3. Show that the linear equation system **LUx=b**, where **L** and **U** are lower and upper triangular matrices respectively, can be solved in two steps: Step 1: **Ly=b**; Step 2: **Ux=y.** Show that both steps can be solved by successive substitution.
4. Show that any matrix **A** expressed as **A=QR,** where **Q** is an orthogonal matrix and **R** is upper triangular. (Hint: Gram-Schmid orthogonalization)
5. Show that the linear equation system **QRx=b** can be solved by $\mathbf{Rx=Q^Tb}$ using successive substitution.
6. Show that $tr\{\boldsymbol{A}\}=\sum \lambda_k$ where λ_k is the k'th eigenvalue of matrix **A.**
7. Show $\det(\mathrm{A})=\prod \lambda_k$ where λ_k is the k'th eigenvalue of matrix **A.**
8. Show that similar matrices have the same eigenvalues. **A** and **B** are similar if there is an **M** matrix such that $\mathbf{A=MBM^{-1}}$. (Hint: Write characteristic equation)
9. Show that symmetric matrices have unit magnitude eigenvalues.
10. Show that symmetric matrices have orthogonal eigenvectors.
11. Use $|\mathbf{AB}|=|\mathbf{A}||\mathbf{B}|$ to establish $|\mathbf{I+ABA^{-1}}|=|\mathbf{B+I}|$. **(|A|=det(A))**
12. Establish the identity tr{**AB**}=tr{**BA**}. (Hint: Use summation definition of matrix multiplication).
13. Establish the identity $|\mathbf{I+AB}|=|\mathbf{I+BA}|$. (Not very easy)
14. Show that distinct eigenvectors of a matrix are linearly independent.
15. Show that singular matrices have at least one eigenvalue with the value zero.
16. Show that eigenvector with zero eigenvalue is orthogonal to eigenvectors with non-zero eigenvalue.
17. Show that eigenvectors of a matrix with zero eigenvalue form the null space, and eigenvectors with non-zero eigenvalue form the range space.
18. Show that range space is orthogonal to null space. (Hint: Use 16,17)

19. Show that matrices with the same set of eigenvectors but with different eigenvalues commute. (Two matrices are said to commute if **AB=BA)**
20. Show **AB=BA** then **A** and **B** have a common set of eigenvectors. (This is much more difficult than 19)
21. Show that **A+ αI** has the eigenvalues of λ_k + **α,** where λ_k is the *k*'th eigenvalue of matrix **A.**
22. Show that $\mathbf{AA^H}$ and $\mathbf{A^HA}$ have the same set of eigenvalues.
23. A less known matrix multiplication matrix is summation of rank-1 matrices. If **C=AB**, then $\sum a_k^c b_k^r$ where a_k^c is column vector and it is the k'th column of matrix **A;** b_k^r is a row vector and it is the k'th row of matrix **B.**
24. Show that $\mathbf{A=M \Lambda M^H}$ where $\mathbf{M=[m_1\ m_2 \dots m_N]}$ ($\mathbf{m_k}$ represent k'th column of matrix **M),** $\mathbf{\Lambda=diag(\lambda_1, \lambda_2,\dots, \lambda_N)}$ is **A=** $\sum_{k=1}^{N} \lambda_k m_k m_k^H$. (Hint: Use 23)
25. Show that if $\mathbf{A=M \Lambda M^H}$ then $A^{-1}=\sum_{k=1}^{N} \frac{1}{\lambda_k} m_k m_k^H$ (Hint: Use 24)
26. Show that if *f(x)* is a polynomial in, and $\mathbf{A=M \Lambda M^{-1}}$, $\mathbf{\Lambda=diag(\lambda_1, \lambda_2,\dots, \lambda_N)}$ **t**hen f(**A)=M f(Λ) M**$^{-1}$ where **f(Λ) =diag(f(λ**$_1$**), f(λ**$_2$**),…, f(λ**$_N$**)).**
27. Solve Hayes 2.4
28. Solve Hayes 2.15
29. Solve Hayes 2.17
30. If **P** is an orthogonal projector and α_1, α_2 are non-zero real numbers; then $(\alpha_1)^2$**P** + $(\alpha_2)^2$ **(I-P)** is invertible.
31. If **S** is real and skew-symmetric, then show that **I+S** is non-singular and the Cayley transform $\mathbf{T=(I-S)(I+S)^{-1}}$ is orthogonal.
32. If **T** is real orthogonal matrix and **(I+T)** is non-singular, prove that we can write $\mathbf{T=(I-S)(I+S)^{-1}}$ where **S** is a skew-symmetric matrix.