

## THE ET INTERVIEW: PROFESSOR C.R. RAO

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Professor C.R. Rao.

It was June 1940. The Second World War was already raging in full swing in its devastation. A young man not yet 20 set out on a 500-mile train journey from the coastal city of Vishakapatnam, India, to Calcutta, the second largest city of the British Empire, after obtaining a first-class first degree in mathematics and with a glimmer of hope of finding a job in the military. The young

man was not so lucky; he was deemed too young for the job. However, while in Calcutta, through a chance encounter, he visited the Indian Statistical Institute (ISI) founded in 1931 by Professor Prasanta Chandra Mahalanobis, a Cambridge-trained physicist. As a last resort he applied for a one-year training program in statistics there. Very promptly he received a positive reply from Professor Mahalanobis admitting him to the Training Section of the ISI from January 1, 1941. This young man was none other than perhaps the most well known statistician in the world today, Calyampudi Radhakrishna Rao (or simply Dr. Rao, as he was universally known to his students and colleagues at the ISI. The "professor" title was reserved for Mahalanobis). Such was the beginning of his spectacular career in statistics, a field chosen as a last resort, and he never looked back. As we will see, time and again this chance factor played a prominent role in Rao's life and work. After an unconventional mix of study and work at the ISI, he was deputed to Cambridge University to apply Mahalanobis distance analysis on some anthropometric data collected by the University Museum. While in Cambridge, he also completed his Ph.D. under R.A. Fisher, the father of modern statistics. By this time Rao had already done some of the most influential work that carries his name, including the Cramér–Rao inequality, the Rao–Blackwell theorem, Rao's score test, and Rao's orthogonal arrays. Perhaps more important for statistical development in independent India, Rao's association with the ISI for the next 30 years has also had a long-lasting effect. After almost 40 years of work, at an age when other people contemplate a relaxed retirement, Rao found himself in the United States starting a new academic life in 1979 on the faculty of the University of Pittsburgh. In 1988 he moved to the Pennsylvania State University. He officially retired from his position at Penn State as the holder of Eberly Family Chair in Statistics only recently at the age of 81.

Dr. Rao has won numerous awards and honors during his distinguished career. Just to mention a few, he is a Fellow of the Royal Society, U.K., and a Member of the National Academy of Sciences, U.S.A. He has been honored with 27 doctoral degrees from universities in 16 countries around the world. In 2001, he received the second highest civilian award, Padma Vibhushan, from the government of India for outstanding contributions to statistics. On June 12, 2002, he received the most coveted award from the president of the United States, the National Medal of Science, with the citation "for his pioneering contributions to the foundations of statistical theory and multivariate statistical methodology and their applications, enriching the physical, biological, mathematical, economic and engineering sciences." Given his more than three score years (1941–2002) of pioneering research in statistics, we can simply describe Dr. Rao as a living legend in statistical science. His work has influenced not only the field of statistics but also has had a profound impact on numerous other areas, such as economics, engineering, agriculture, anthropology, biometry, demography, genetics, geology, and medicine.

In October 2001, I traveled to State College to conduct this interview and enjoyed the kind hospitality of Dr. and Mrs. Rao. After looking at some of my sample questions, Dr. Rao preferred to respond in writing due to the sensitive nature of some historical facts. Though we lost some of the flexibility of a conversational dialogue, what we now have, I believe, is a well-thought-out social, historical, and academic document of a vanishing world to which we

are still connected by living memory only through Dr. Rao. He attained the age of 82 on September 10, 2002. He is still active in research, fulfilling his quota of four or five published research papers per year. We wish him a long and productive life to enrich the field of statistics further.

While conducting and compiling this interview, I was fortunate to have a great deal of help from my colleagues. I am most thankful to them. I am most grateful to my student Aurobindo Ghosh, who provided immense help throughout this project and gave many thoughtful comments and pertinent suggestions. I am also grateful to my secretary, Naoko Miki; to Janet Fitch; and to Mary Timmins, Brett Graham, and Luis Galvis for their invaluable assistance. Finally, I would like to acknowledge the encouragement and guidance of Professor Jayanta Ghosh, former director of the ISI. He generously shared his deep understanding of the finer points in statistics and his first-hand knowledge of Professor Mahalanobis, Dr. Rao, and the ISI.

#### **FAMILY BACKGROUND, EARLY EDUCATION, AND COLLEGE: 1920–1940**

Welcome, Dr. Rao. I am really grateful to you for agreeing to do this interview. Perhaps we can start with your family background and early childhood. You grew up in a family of six boys and four girls. What was it like growing up in a large family? Could you explain the significance of your given name Radhakrishna? Also, could you reflect upon the caste system in those days in your area and how that affected you?

My father, C.D. Naidu (1879–1940), worked in the police department under British rule and had the designation of inspector of police when he retired at the mandatory age of 55. My mother, A. Laxmikantamma, was probably 20 years old when she got married; she bore 10 children over a period of 17 years. There was no concept of family planning in those days, and it was not unusual for a couple to raise a large family.

The first child was a daughter, Laxmi Narasamma, born in 1909; she was followed by a male child who died young. The second daughter, Sanjeevamma, was born in 1911; she was followed by a male child who also died. The third daughter, Sakuntamma, was born in 1913, followed by a male child born in 1916. Having lost two earlier sons, my parents were prepared to do anything to save the newborn male child. Guided by a superstitious belief, my parents took the newborn child out of the house, placed it on a garbage heap, and brought it back into the house, symbolically portraying the loss of their child and adopting an abandoned one. The ritual seemed to have the desired effect: the child survived! He was named Thippanna, which literally means “garbage heap” in one of the South Indian languages. Thippanna was followed by three sons, Venkateswara in 1917, Radhakrishna (myself) in 1920, Ramachandra in 1922, and a daughter, Neelavati, in 1925. The appendage Rao was added to all the names of male children in accordance with the custom in some communities of South India of adding such names as Naidu, Rao, Swamy, etc., to given names. (The

last name Rao in my name is actually a part of my given name.) Generally, the boys are given the names of gods of the Hindu Pantheon, like Venkateswara, Radhakrishna, and Ramachandra were given to the last three sons by my parents. But my name, Radhakrishna, has a special meaning. I was the eighth child and, following the usual custom, was named after the god Krishna, who was the eighth child of his parents. Krishna was a romantic person, and one day he eloped with his married aunt Radha. The word Radhakrishna was coined to symbolize the highest form of romance and as an alternative name for Krishna. I am romantic by temperament, but all my romance is in the wrong place, in the head and not in my heart.

We did not have any particular problem in growing up in a large family. My father was away from the house on duty most of the time, and we saw very little of him. My mother was a great disciplinarian, and she controlled our daily activities, prescribing the time for playing, studying, and sleeping. Perhaps this regimen helped us in leading disciplined and successful lives. My elder sisters did not go to school, as it was not the custom to educate girls. Instead they helped my mother in all the household tasks and in looking after the younger brothers and sisters.

In my younger days the caste system was quite rigid, and even now it exists in some form or other. The Brahmins, with the highest ranking in the caste hierarchy of the Hindu society, were prominent in all activities and tried to distance themselves from the other castes, the rarest form of which were untouchability and unseeability practiced in the Travancore region of South India. (Untouchability meant that a person of a lower caste was not allowed to touch a Brahmin. Unseeability meant that a lower caste person had to hide himself if a Brahmin was in the vicinity.) Most of my schoolmates were Brahmins, and I belonged to one of the lower castes, known as Padma Velama, a community of landlords. I could play with my schoolmates at school and on the playground without the knowledge of their parents but could not visit them at their houses. Lower caste people were not allowed inside the homes of Brahmins. As a bright student with a good school record, I was considered worthy enough to go up to the courtyard of a Brahmin's house but not to enter it. If I were thirsty while walking back with a Brahmin friend from the playground and needed a drink of water, my friend's family would not serve me water in a glass, as they have to discard the glass as polluted once I touched it. If I wanted to drink water, I would have to stand at a distance, with my two hands cupped together in front of them to receive the water poured into my hands from a height. My family experienced difficulties renting houses in predominantly Brahmin localities.

There was another incident where the caste system showed up in an ugly way. The head of the Mathematics Department at Andhra University where I studied, Professor V. Ramaswami, a Ph.D. from Cambridge University, was a non-Brahmin by caste, like me. When I obtained the first rank in first class in the final M.A. examination in mathematics at Andhra University, the friends of two of my classmates who were Brahmins and who were competing with me for the first rank started a rumor that the head of the department had showed

favoritism in grading my answers because I was a non-Brahmin. I was unhappy when I heard about it but took it as something natural in the Indian society, where casteism, provincialism, and linguistic affinities play a dominant role in human relations.

Throughout your elementary and high school years, your family had to move frequently, due to your father's job as an inspector of police. These moves certainly did not affect your academic performance; you secured the first position in all grades. Were you affected in other ways?

Yes, my father was transferred every two or three years from one place to another. As a result of these frequent moves, I studied in several different schools and completed grades 2 and 3 in Gudur, 4 and 5 in Nuzvid, and 6 and 7 in Nandigama, all in the state of Andhra Pradesh. The subjects and the levels at which they were taught were not very different from school to school in different places, and there was no difficulty in adjusting to a new place. At this stage, my father retired and settled down in Vishakapatnam, a coastal city in Andhra Pradesh. It is a well-known port with extensive beaches and surrounded by mountains. The main consideration in moving to Vishakapatnam was the existence of good educational facilities, including high schools, Andhra University, and a medical college, which could meet the educational needs of the children. In this city my brothers and I studied continuously over a period of 10 years. I completed my high school, intermediate, and university education, ultimately receiving my first university degree, the Bachelor of Arts with Honors in Mathematics, which was automatically upgraded to master's degree. My elder brothers, Thippanna and Venkateswara, completed B.Com and a medical degree, respectively. My younger brother was sent to Madras to study engineering after he completed his intermediate degree, which was a two-year course after high school where students were introduced to advanced topics in subjects of their choice.

Did you have any inspiring teachers of mathematics? What really led you to do your undergraduate degree in mathematics (honors)? Were you advised to do this by your family?

From my earliest years, I had an interest in mathematics. In the elementary school, at the age of six, I knew by heart the multiplication tables up to 20 by 20. I recall the teacher of my class lining up the boys in the class and asking me to stand in front of them and recite the multiplication tables line by line, which the other boys would then repeat. When I was 11, I could do complicated arithmetical problems without paper and pencil. My father appreciated my interest in mathematics and my good performance in school, and he thought that I should eventually get a degree in mathematics and proceed to do research to get a doctorate degree. He presented me with a book called *Problems for Leelavathi*, a collection of problems set by a mathematician for his daughter Leelavathi to solve. He asked me to work out 5 to 10 problems in the book every day. I enjoyed solving these problems, which aroused further interest in

me to pursue mathematics. Thus, my entry into mathematics resulted from the encouragement I received from my father and my own interest in solving mathematical problems. I do not remember any particular role played by the school-teachers in my pursuit of mathematics.

In the intermediate class (after high school) you won the Chandrasekara Iyer Scholarship in Physics, named after the father of the famous physicist and Nobel laureate Chandrasekar V. Raman. C.V. Raman also attended the same college you did, Mrs. A.V.N. College in Vishakapatnam. Can you tell us more about this and other highlights from your early academic career?

After completing high school (11th grade), I joined the two-year intermediate course, before entering Andhra University, at the Mrs. A.V.N. College. In the intermediate course, in addition to the compulsory subjects, English and a local language, one had the option of choosing three science subjects for specialization. I chose mathematics, physics, and chemistry, which would make me eligible to pursue higher studies in any one of these subjects at the university.

C.V. Raman, who won the Nobel Prize for his invention of spectroscopy, was an alumnus of Mrs. A.V.N. College, and he instituted a scholarship in memory of his father, Chandrasekara Iyer, to be awarded to a student in each year of the intermediate class who stood first in a competitive examination in physics. I won the scholarship, which carries a cash prize, in both years of the intermediate class. The college magazine issued in 1935 carried my photo with the following caption:

C. Radhakrishna Rao who has won the Chandrasekara Scholarship this year. He has had the unique distinction of knocking off the most coveted prizes throughout his school career.

The two-year period in the intermediate class was an exciting one in my educational career. Winning the Chandrasekara Scholarship for two consecutive years gave me confidence to continue my academic pursuits, even with uncertain job opportunities. Although I developed an aptitude for physics in preparing for the competitive examinations for the Chandrasekara Scholarship, I decided finally to stick to mathematics, which was my father's wish.

You mentioned your father's role in developing your interest in mathematics. Can you say more on the influence of your parents? What did you inherit from them? Elsewhere you have recorded the influence of your mother. In the dedication in your book *Statistics and Truth: Putting Chance to Work*, you credited your mother "For instilling the quest of knowledge" in you and "who woke you up every day at four in the morning and lit the oil lamp to study in the quiet hours of morning when the mind is fresh."

My mother was a stern disciplinarian. She set up a daily routine for play and work for the boys—go out to play at 4 P.M. and come back at 6 P.M., study for two to three hours and go to bed, wake up at 4 A.M. and study till 6 A.M.



At age 15: Published in the Mrs. A.V.N. College magazine.

We had very little contact with our father until he retired. He would even forget the order in which the eighth and ninth children were born. After retirement and moving to Vishakapatnam, he started investigating what each son was doing and guiding our activities. He was happy that I was showing good progress in my studies and would often refer to me as his “pride, hope and joy.” His ambition was to see me as a full-fledged mathematician with a Ph.D. Unfortunately, my father died a few months before I completed my master’s degree in mathematics in first class with first rank. As my father desired, I chose research as my primary occupation and obtained Ph.D. and Sc.D. degrees from Cambridge University. I never expected that I would accumulate 27 honorary doctorates from universities in 16 countries around the world—he would have enjoyed seeing me in academic robes of different colors, receiving those diplomas.

We were, indeed, very fortunate in having parents who fostered our innate abilities with proper guidance, provided an environment conducive to study, and gave us a framework for ethics of life and work ethics to enable us to achieve higher ideals in life. I may also add that, perhaps, genetics played an important role in whatever I have achieved in my life. I must have inherited my father’s analytical ability and my mother’s tenacity and industry.

My father attached great importance to education and scholastic achievement, especially because he had ended his own education after passing the intermediate examination and had taken a job as a policeman. He had done this according to the then-prevalent norm of acquiring only the minimum qualifica-

tions needed to get a job and also because he needed to be an independent earning member, as his parents were not well off.

Raising a large family and educating the children was a great financial obligation for my parents. In spite of that, my father also adopted poor boys not related to our family and paid for their education and living expenses, as well.

In memory of the sacrifices made by my parents in educating me and my brothers and preparing us for successful careers, I made a donation to Andhra University to institute a gold medal award in the name of my parents to be given every year to the student graduating with the first rank in the master's degree program in statistics from the university.

#### **AT THE INDIAN STATISTICAL INSTITUTE (ISI): 1941-1946**

In my introduction I described briefly how you joined the ISI as a "statistics trainee" by a chance encounter. Can you say more on that accidental meeting with Mr. Subramanyam and visit to the ISI, which changed your life?

I was less than 20 years of age when I completed my university education and received a degree equivalent to that of a master's in mathematics. I wanted to pursue a research career and hoped that my good performance in the examinations at the university would enable me to get a research scholarship. Due to some administrative formalities, my application for a research scholarship at Andhra University was turned down. There was pressure from my family to sit for competitive exams for the Indian Civil Service (ICS), which was the route to a lucrative job and the chance of marrying an accomplished girl from a wealthy community. But I was underage and had to wait for about 18 months before taking the exams. So I decided to take up a job during the waiting period. I responded to an advertisement for a mathematician to work in an army survey unit. I was called for an interview in Calcutta. I did not get the job, but I found something that would keep me engaged for the rest of my life.

In Calcutta, I was staying in a South Indian hotel where I met, by chance, a young South Indian named Subramanyam. He told me that he was employed in Bombay and he had been sent to Calcutta for "training in statistics" at the ISI, the existence of which was unknown to me at that time. I had taken a course on probability and statistics as an optional subject for my master's degree in mathematics at Andhra University, and I was curious to know about the ISI and its programs. Subramanyam took me to the ISI, which at that time was located in a few rooms in the Physics Laboratory of the Presidency College. He said that the ISI had been started by P.C. Mahalanobis, professor of Physics; ISI had no premises of its own, and all its activities were conducted from a few rooms in the Physics Laboratory. He said, "The ISI offers a one-year training program in statistics, and there are good job opportunities for those who have a certificate of training from the ISI." He also showed me a research paper he had written. With my knowledge of probability, I could understand his work. I



thought, by getting admitted to the training program at the ISI, I could achieve the twin objectives of getting a job and also testing my abilities to do research. With a letter of recommendation to Professor P.C. Mahalanobis from V.S. Krishna, vice-chancellor of Andhra University, I was admitted to the training program at the ISI.

Your father had passed away the previous year. How could you afford to study in Calcutta?

I was to join the ISI by early January of 1941. There were, however, some problems to be faced. The first was to convince my mother that the ISI program would give me an additional qualification for getting a job and also for faring better in the competitive exam for ICS. The second was to look for financial support for study in Calcutta, as there was no regular family income after my father's death. I needed Rs. 50 (approximately a little more than \$1, at the current exchange rate) for my admission fee and Rs. 30 per month for board and lodging in Calcutta. One of my elder brothers, who was employed by then, agreed to pay Rs. 12 per month, and my mother provided the rest of the money from the modest investments she made when my father was alive. Thus, the stage was set for my journey from Vishakapatnam to Calcutta for a one-year stay. I paid the admission fee of Rs. 50 to the ISI and stayed in a nearby hotel, where the lodging cost was Rs. 12 and boarding, Rs. 13, and I still had Rs. 5 per month left for other expenses.

What was the nature of the "one-year statistical training" course? Did Professor Mahalanobis teach any subject? Did you see him much during this period?

The one-year training program at the ISI was not up to my expectations. The courses were more descriptive in nature, and they were given by teachers who had no expertise in theoretical statistics. The well-known statisticians R.C. Bose, S.N. Roy, and K.R. Nair who were then working at ISI were not involved in teaching. I did not learn much from the courses in the training program, but I had the opportunity to get in touch with Bose and Nair and follow their work on combinatorial mathematics used in experimental designs. I started doing research in the design of experiments and writing papers jointly with Nair. My first paper with Nair appeared as a short communication in 1941, three months after I joined the ISI. This was followed by several papers written with Nair in 1942.

The Professor, as Professor Mahalanobis was referred to by everyone in the ISI and elsewhere, did not give any lectures. We had no opportunity even to see him or talk to him. He had no office hours. He would come at any time and walk directly to his office. The "workers" of the ISI, as all staff members were called, seemed to be on alert so long as he was in the office. He could call any worker to his office by sending a message through his personal attendant for what was described in the diaries kept by the "workers" as "discussion with the Professor." The workers seemed to be afraid of the Professor and carried out

all the decisions made by him in all matters. I had the impression that the workers were more loyal to the Professor than to the institution in which they were working. Later, when I became the head of the Research and Training School in the ISI, I would invoke Professor's name to get things done quickly and without any opposition. Perhaps an autocrat like him was necessary to achieve what he did for the development of statistics and placing India not far from the center of the statistical map of the world.

During your one-year trainee period, you decided to enroll in the master's degree program in statistics that just started in July 1941 at Calcutta University. What prompted you to do that? And could you carry on research work while being a master's degree student? In 1943 you obtained an M.A. in statistics with a first class, securing the first rank with marks of 87.5%, still a record at Calcutta University. Could you comment on that achievement?

I joined the ISI in the beginning of 1941 and was happy with what I was doing by way of learning statistics by studying books and journals and doing research. A few months later, Calcutta University announced the launch of a new master's degree program in statistics, the first university in India to do so. The Professor was chosen as the head of the Department of Statistics with R.C. Bose, S.N. Roy, K.R. Nair, and A.K. Bhattacharya as part-time lecturers. The Professor suggested that I should enroll and get a master's degree. I thought this would be a nice qualification, and so I was admitted as a student at Calcutta University. I continued to do research while being a student and sometimes felt that it was waste of time to attend classes and prepare for tests. However, I was glad that I acquired a degree, the first M.A. degree in statistics from any university outside the U.K., especially because I secured the first class with record marks and won the gold medal.

Who were your classmates and teachers during your M.A. program? What were the courses? Looking back now, what were the major influences on you?

There were only five or six students in the master's program. Of them only one, Harikinkar Nandi from Calcutta, was competing with me for the first position. His friends were disappointed that he got the second rank, especially that someone who was not a "son of the soil" stole the first rank. Having lived in different linguistic regions of India, I realized that this was a natural feeling.

Of course, we had good teachers, R.C. Bose, S.N. Roy, K.R. Nair, and S. Sengupta. The most inspiring was R.C. Bose. I owe much to him for my early research work in statistics. He was teaching linear models and design of experiments. He introduced the concept of nonestimability of a parametric function, which led me to develop the concept of generalized inverse of a matrix and provide a general framework for the treatment of linear models. I also followed up his work on combinatorics of design of experiments and developed new incomplete block designs for varietal trials and fractional replications for fac-

No. 1893

Calcutta University



SENATE HOUSE:

The 30.2.1943

M.A. Marks.

The following are the marks obtained by C. Radhakrishna Rao  
 No. Calcutta of Presidency College at the M.A. Examination of  
 1943 in Statistics

Practical										
Final Paper	Second Paper	Third Paper	Fourth Paper	Fifth Paper	Sixth Paper	Seventh Paper	Eighth Paper	Ninth Paper	Tenth Paper	Eleventh Paper
90	87	81	82	92	432	83	140	45	268	700

Rs. 3.

Prepared by—

*[Signature]*

*[Signature]*  
 Controller of Examinations.

Mark sheet of the M.A. degree in statistics with record marks, 1943.

torial experiments through orthogonal arrays that are widely used in industrial experimentation. I also collaborated with R.C. Bose and the famous number theorist S.D. Chowla and wrote a few joint papers on combinatorial mathematics.

S.N. Roy taught multivariate analysis, testing of hypotheses, and estimation. He used geometrical concepts in deriving distributions of sample statistics, which I later used in deriving a test, now called Rao's *U*-statistic, for redundancy of a set of measurements in multivariate analysis. R.C. Bose and S.N. Roy were thus most instrumental for my early contributions to two different areas of statistics, design of experiments and multivariate analysis.

As there were no textbooks on statistics, the teachers used to read original papers from journals and present the content to us in the class. I remember one incident. After finishing the courses, we had what was called preparatory holidays before the final examination. While I was reading the notes on testing of hypotheses given by S.N. Roy, I discovered some statements that needed modification. I met S.N. Roy and told him about it. He agreed with me and recalled the students from their holidays for a special class to make the amendments.

I remember attending only one lecture given by Professor Mahalanobis, on what he called the scrutiny of data—how to detect errors in recorded data. This was interesting and useful. R.A. Fisher, with whom I worked for my Ph.D., also used to emphasize the need for what he called “cross-examination of data” before applying sophisticated statistical analysis. I have followed the advice of my two mentors, Mahalanobis and Fisher, in analyzing large sets of anthropometric data. In one case, I recommended the rejection of an entire set of survey data after scrutiny. In my book *Statistics and Truth: Putting Chance to Work* [11], I have given some examples where statistical analysis carried out on unscrutinized data led to possibly misleading conclusions.

I was lucky to come under the influence of such stalwarts as Fisher, Mahalanobis, Bose, and Roy, who helped me realize that statistical theory and practice are intertwined and that one cannot exist without the other.

By 1943 you already had quite a few publications. How did you really get started with research so quickly? For your M.A. degree in statistics, you chose to write a thesis in lieu of two practical papers for the final examination. In this thesis you proved a characterization problem posed by Ragnar Frisch, which was later published in *Econometrica* [49]. Can you tell us more about your master’s thesis?

As I have mentioned earlier, I started working on the design of experiments, as that was the most popular subject at the ISI when I went there in 1941. It was a subject that did not depend on the knowledge of statistical theory or probability. I continued to do research on the design of experiments and some problems in multivariate analysis on my own while taking courses for my M.A. degree. By the end of 1943, before I completed the M.A. course, I had written some papers, which were already published, and a few others had been presented at conferences or submitted for publication. For the M.A. examination, we had eight papers, six of which were on statistical theory and two on practical problems. There was an option to submit a thesis in lieu of the two practical papers, and in view of the research work I had already done, I exercised this option.

Looking through my thesis, forwarded by Professor P.C. Mahalanobis on June 18, 1943, to the controller of examinations of Calcutta University, I find that I must have been working hard during the period 1941–1943. The thesis was in three parts, the first with 119 pages on design of experiments, the second with 28 pages on multivariate tests, and the third with 42 pages on bivariate distributions. The second part was devoted to what I called a perimeter test, which is the same as the Hotelling–Lawley trace test, introduced a few years later. This test was presented at a session of the Indian Science Congress in 1943. In the third part, I offered a solution to a problem raised by Ragnar Frisch: a paper on this was published in *Econometrica* in 1947 [49]. I believe that Hotelling was a referee of this paper. I saw this problem mentioned in a paper by Allen while browsing through the publication *Statistical Research Memoirs* (1938), edited by J. Neyman. I was actually reading a paper by Neyman and

David in this volume. The paper by Allen attracted my attention and created an interest in characterization problems. I had the opportunity to develop this area in collaboration with the Russian statistician Linnik, my student Laha, and my colleagues Khatri and Shanbhag. A number of books and numerous research papers have appeared on this subject during the last 50 years.

In a letter dated December 16, 1991, to S.P. Mukherjee, head of the Statistics Department of Calcutta University, K.R. Nair mentioned the following about my M.A. thesis.

The late Professor K.B. Madhava and the late Professor U.S. Nair were external examiners for the final year M.Sc. Examination of 1943. On going through Rao's dissertation, U.S. Nair remarked to me that it was of a very high class, almost equivalent to a Ph.D. degree.

Most of the results given in the thesis have been published elsewhere. The thesis reflects my early interests in four areas of statistics: the design of experiments, linear models, multivariate analysis, and the characterization of probability distributions. These kept me engaged for the next 60 years.

My next question is somewhat long winded. It is about the research you did after your master's degree. Professor Mahalanobis offered jobs as technical apprentices to all of you who passed the M.A. examination in the ISI. The years that followed were most eventful for you. During this period, you wrote a paper [33] that was more of an "overnight adventure" that had everything on it: Cramér-Rao bound, Rao-Blackwellization, differential geometry for the first time in the statistics literature, and Fisher-Rao metric (possibly I am still missing something!). I think econometricians will be delighted to hear the story of this paper directly from you. What other research did you do during this period? Possibly, all of that work got overshadowed by the Cramér-Rao lower-bound paper. You wrote a one-page note with S. Janardhan Poti [47], which I consider to be the precursor to your celebrated score test principle. Any comments on this "little note"?

Yes, the Professor offered jobs to three of us, H.K. Nandi, N.K. Chakravarty, and myself, who passed M.A. in first class. We were called T.A.s (technical apprentices), and the salary offered was Rs. 75 a month. I had also an offer of a part-time lectureship at Calcutta University for Rs. 100 a month. However, a few unpleasant things happened. A few days after we joined the ISI, Nandi and Chakravarty resigned in protest because the Professor had made, I was told, "some insulting remarks" about Chakravarty. I was not sure whether they expected me to resign also. They did not talk to me about it.

I too had a small encounter with the Professor. I had two first-class master's degrees, one in mathematics and another in statistics, and expected a higher salary. One day, I worked up the courage to tell him that Rs. 75 was not enough.

He responded, "You are asking for more! Do not be a fool. You will hear from me soon." I withdrew from his office shaking in my boots. I was glad to receive a letter a few days later offering me Rs. 150 a month, a 100% increase!

I started working at the ISI as a regular employee in November 1943 and as a part-time lecturer at Calcutta University in June 1944. The period from January 1944 to July 1946, before I went to Cambridge, was, perhaps, the most eventful of my research career.

I continued my research on combinatorics with reference to design of experiments and wrote a number of papers, some jointly with R.C. Bose and S.D. Chowla. I developed a general theory of least squares without any assumptions on the concomitant variables. I found a test for redundancy of a specified set of variables in multivariate analysis. S.J. Poti, who joined the ISI as a T.A. in 1944, and I were sharing an apartment in Calcutta. We used to go for long walks in the evening, often discussing problems in statistics. One day he asked me whether the Neyman–Pearson theory could be used to test a hypothesis about a parameter when the alternatives are one-sided. I said "of course" and gave him an immediate solution. This was published as a note jointly with Poti [47]. An extension of this to the multiparameter case is the score test, the details of which appeared in 1948 [58]. The motivation for introducing the score test was somewhat different from the problem posed by Poti. I was interested in testing whether different estimates of a parameter computed from different data sets are consistent with each other, a problem that arose in pooling estimates of linkage parameters in genetic studies, obtained from different experiments.

All of my contributions in combinatorics, linear estimation, and multivariate analysis were overshadowed by a simple bound I found for the variance of an unbiased estimate of a parameter. This was named Cramér–Rao inequality by Neyman in one of his papers published in 1947, and it appears as a "scientific and technical term" in *McGraw-Hill Dictionary of Scientific and Technical Terms* (fifth edition, p. 476). The paper in which the Cramér–Rao inequality was given was included in the publication, *Breakthroughs in Statistics: 1890–1990* edited by Kotz and Johnson and published by Springer-Verlag.

It may be of interest to younger researchers to know about the origin of this paper. I was giving a course on estimation to the senior students of the master's class at Calcutta University in 1944, where I mentioned without proof Fisher's information inequality for the asymptotic variance of a consistent estimate. There was a bright student in the class, V.M. Dandekar, who raised the question whether such an inequality exists for the exact variance of an estimate in small samples. I said I would try and let them know. That night, I read Fisher's papers but could not get any clue, as Fisher did not give a satisfactory proof of his inequality. I began to think about the problem independently. I tried for perhaps a couple of hours. Assuming that an estimate  $T$  is unbiased for a parametric function  $\Phi(\theta)$ , it was easy to show that  $\Phi'(\theta)$  is the covariance between  $T$  and  $(1/f) df/d\theta$ , where  $f$  is the density function of the observations. Then, an

application of the Cauchy–Schwartz inequality, or the fact that the correlation coefficient does not exceed unity in modulus, gives the desired inequality

$$V(T)V(1/f df/d\theta) \geq [\Phi'(\theta)]^2, \quad \text{or} \quad V(T)I(\theta) \geq [\Phi'(\theta)]^2,$$

where  $I(\theta)$  is information defined by Fisher. I was a little surprised at the simplicity of the result but also felt glad that it was not too complicated to discuss in the classroom. I proved the result in the class in my next lecture.

While writing a note on this result for publication, I discovered various related results. First was an extension to the multiparameter case. Second was the use of a sufficient statistic in improving the efficiency of an estimate (a result named the Rao–Blackwell theorem by Lehmann and Scheffé and a process termed Rao–Blackwellization by Berkson). Third was the Fisher–Rao metric and Rao distance based on differential geometric concepts. All of these results were given in my paper [33] in 1945. (The paper was written in early 1944, but publication of *Sankhyā*, where it should have appeared, was suspended during the Second World War, and I had to submit it to a mathematics journal.) I did not realize that the paper would receive wide attention and would generate research on various aspects of estimation in small samples and differential geometric approach in statistical inference. Some of the results in the paper would one day appear in most of the books on mathematical statistics.

I derived a few other results, including those known as Bhattacharya bounds. I sent a note on the extended results to *Annals of Mathematical Statistics*, which was accepted for publication. By that time I was in Cambridge and heard that



With David Blackwell, 1977.

Bhattacharya working independently obtained similar results and got them published in *Sankhyā*. I was in a dilemma and decided to withdraw my paper.

I may mention some anecdotes in connection with the Rao–Blackwell theorem and Rao–Blackwellization. Blackwell independently discovered this method two years after the publication of my paper. In 1953, I attended a conference where Berkson presented a paper using conditional expectation with respect to a sufficient statistic to improve an estimate, attributing the method to Blackwell and calling the method Blackwellization. When I told him that the method was first given by me, Berkson remarked that Raoization by itself does not sound nice; in a later paper I found that Berkson used the term Rao–Blackwellization. Another instance was a review of a book by Lindley, where he attributed my result on the use of a sufficient statistic to Blackwell only. When I wrote to him about my priority of the result, he replied, “Yes, I read your paper. Although the result was in your paper, you did not realize its importance because you did not mention it in the introduction to your paper.” I replied to Lindley saying that it was my first full-length paper and I did not know that the introduction to a paper is written for the benefit of those who read only the introduction and do not go through the paper.

What other activities were you engaged in besides teaching and your own research? By 1946, when you had settled down to life at ISI, destiny again took over, and the course of events of your life changed because of a cable from Cambridge. Can you describe that turn of events?

When I joined the ISI as a T.A., Mahalanobis assigned me two projects. One was to assist him in the editorial work of *Sankhyā*, the *Indian Journal of Statistics*, which he had started in 1933. As a registered society, the ISI could have its own journal and make its own arrangements for publishing it. But for some reason, which might have been valid at the time *Sankhyā* was started, the Professor had created the Statistical Publishing Society (in 1935) and had given it the responsibility for bringing out *Sankhyā*. He also acquired a press with all the equipment needed for printing mathematical papers and installed it in his family house. The press was later expanded and called Eka Press in 1938. This press was used to print *Sankhyā* until 1995. The compositors often had difficulties in typesetting the mathematical formulas. I remember that in addition to the editorial work of *Sankhyā* of receiving papers and getting them refereed, I would go to the press to help the compositors in putting the mathematical symbols in the correct places. I continued to do the editorial work for *Sankhyā* in an unofficial capacity for a period of about 20 years. In 1964, I was named an associate editor of *Sankhyā*, and I took over full responsibility as the editor after the death of the Professor in 1972.

The second project was more challenging—to take charge of an anthropometric project. During the population census of 1941, D.N. Mazumdar, a well-known professor of anthropology at Lucknow University, took between 9 and



11 anthropometrics measurements on each of about 4,000 individuals belonging to 23 castes and tribes in the Indian state of Uttar Pradesh. The ISI was asked to acquire the data and do appropriate statistical analysis. Mahalanobis said that it would be a good experience for me to take charge of the data, do the analysis, and write a report. I accepted the project for several reasons.

I had read about Mahalanobis  $D^2$  distance, a measure of dissimilarity between two populations based on multiple measurements. I wanted to see how it worked in practice and what significant conclusions could be drawn.

I believed then, as I do now, that practical problems provide the motivation for forging new statistical methods and thus contribute to the expansion of statistics. In fact, while working on anthropometric data, I developed a test, known as Rao's  $U$ -statistic, for redundancy of certain measurements and the concept of canonical coordinates for graphical representation of high-dimensional data in a low-dimensional space. Canonical coordinates are the forerunner of multidimensional scaling and correspondence analysis. I generalized analysis of variance (ANOVA) to multiple measurements and called it analysis of dispersion (AD). In the United States, it was called multivariate ANOVA (MANOVA). I had also introduced a method of cluster analysis, which was perhaps the first major application of cluster analysis. My first book [2] arose out of my anthropometric work. It has the early use of AD or MANOVA, in addition to special problems related to multiple measurements. This book has a nice mix of theory and applications to real data but went out of circulation as the demand for books on mathematical statistics without applications (considered distractions by some statisticians) arose, especially in the United States.

Finally, I did not want to offend Professor Mahalanobis by saying "no." In retrospect, I think I did a wise thing by saying "yes," as subsequent events showed.

While I was still working on the anthropometric data, Professor Mahalanobis received a cable (in March 1946) from J.C. Trevor, an anthropologist working at the Duckworth Laboratory in the University Museum of Archaeology and Ethnology at Cambridge, United Kingdom. It said that the museum had acquired a large collection of skeletal material from ancient graves in Africa by a British expedition financed by the Wellcome Trust and that he would like to have someone from the ISI analyze the data using the methods developed by Mahalanobis. The Professor was pleased, and he immediately replied that he was deputing two of his students: Rao, who is good in statistics, and Mukherjee, who is good in anthropology. Both of us left for Cambridge in August 1946. We were employed by the museum for about £30 per month for two years. I joined King's College, Cambridge, where Mahalanobis had studied, and also registered for a Ph.D. degree under R.A. Fisher. (In 1974, I was made an Honorary Life Fellow of King's College, an honor that includes walking on the lawns, a privilege denied to students and outsiders. The number of Life Fellows is limited to 11 persons at any one time.) My family was happy that I was going abroad.

**TIME AT CAMBRIDGE: AUGUST 1946–AUGUST 1948**

At Cambridge, you had a full-time job at the Museum of Archaeology and Ethnology and also worked in R.A. Fisher's genetics laboratory. Plus you registered for a Ph.D. degree under Fisher. Did you have any problem adjusting? How would you describe the atmosphere of Cambridge at that time? How was Fisher as a thesis adviser?

Yes, I had a full-time job at the museum. When I asked Fisher to be my thesis supervisor, he agreed on the condition that I spend some time in his genetics laboratory, where he was breeding mice to map the chromosomes. I did not know what I was expected to do. However, I agreed, hoping that some practical experience in genetics would be useful. I used to work in the museum during the day and in Fisher's lab for a few hours in the evening.

I had no problem in adjusting to life in Cambridge. I lived in one of the dorms of King's College. I liked the intellectual atmosphere of the place, attending the debates organized by the Cambridge Debating Society (I had been a member of the debating team in intercollegiate competitions when I was in Vishakapatnam), listening to invited talks by famous scientists and politicians, auditing the series of lectures given by Bertrand Russell, and participating in other student activities. When Bertrand Russell lectured, around 1,000 students



Working with mice in R.A. Fisher's genetics laboratory, 1947.

would show up, and everyone could not be accommodated in one room. You had to go in advance to get a seat in the main lecture hall. Those who sat in other rooms could only hear the lecture on the loudspeaker system. There was no TV those days.

I asked Fisher to suggest a research problem for my Ph.D. thesis. He said the problem must be mine and that he would only advise me if and when I encountered difficulties. This was good advice. I used to say the same thing to my Ph.D. students without success. There were only 2 cases (out of 50), where the students chose their own problems. Fisher was always available for consultation at any time I wanted, and I did not have to make appointments. I did not seek much help from him in writing my thesis but benefited by attending his lectures and seminars in genetics.

Continuing with Fisher, he was a “controversial” figure, but how would you summarize his contribution to statistics?

Fisher is the father of modern statistics. He formulated three methodological aspects of statistics as specification (stochastic model for data), estimation, and testing of hypotheses. He introduced the concepts of sufficiency, efficiency, information, and maximum likelihood in estimation theory. He laid the foundations of the design of experiments. All of these form the main contents of statistics as we study and do research even today. Fisher did not provide rigorous mathematical proofs of the propositions he stated, for which he was unjustly criticized. I believe the deficiencies in Fisher’s proofs are fixed up by various researchers.

Fisher was always generous in giving his ideas to his students. He was involved in many controversies on statistical methodology. Perhaps these are inevitable in a subject like statistics, which is concerned with drawing conclusions under uncertainty.

During your time at Cambridge, you also wrote one of the most influential papers in statistics. “Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation.” As you know, this test is now an essential tool for econometricians. Could you tell a bit about the motivation and history of this test?

You are referring to my score test paper [58]. The test evolved in a natural way while I was analyzing some genetic data. As I recall, the problem was the estimation of a linkage parameter using data sets from different experiments designed in such a way that each data set had information on the same linkage parameter. It was, however, necessary to test whether such an assumption could be made because of unforeseen factors affecting the experiments. This required a test for consistency of estimates derived from different experimental data sets. Nowadays, the methodology used for such tests is called “meta-analysis.” Denoting the log-likelihoods and the information functions of  $k$  parameters by  $L_1(\theta_1), \dots, L_k(\theta_k)$  and  $I_1(\theta_1), \dots, I_k(\theta_k)$ , respectively, I suggested the criterion

$$\sum_{i=1}^k [I_i(\theta_e)]^{-1} (dL_i/d\theta_e)^2$$

for testing the hypothesis  $\theta_1 = \dots = \theta_k$ , where  $\theta_e$  is the maximum likelihood estimate of the common  $\theta$ . This suggested a general criterion for testing hypotheses of the form  $H(\theta_1, \dots, \theta_k) = c$ , given the joint log-likelihood function  $L(\theta_1, \dots, \theta_k)$ , which I called the score test.

I wrote a note on the score test and showed it to Fisher, mentioning that I would use this test in my linkage problem. He refused to read the paper, saying that he would like to see how the test works on real data. He also suggested that I write a paper using my test on some real data, which would be appropriate for publication in his new journal, *Heredity*, and, perhaps another note on the derivation and distribution of the test for publication elsewhere. Fisher was pleased to see the application of my score test in the analysis of linkage data and accepted my paper for publication in *Heredity*. The other paper dealing with the theoretical discussion of the score test was published in the *Proceedings of the Cambridge Philosophical Society* in 1948. The score test is another example of a statistical method motivated by a practical problem.

For a long time this work was not “noticed” by others—even by researchers engaged in the area of testing. Did that surprise you? And how do you feel about the “holy trinity”—likelihood ratio, Wald, and score tests?

I was aware that my paper on the score test did not receive any attention for a long time. It was rediscovered 10 years later by Silvey, who named it the Lagrangian multiplier (LM) test. I conjectured that the score test is good for local alternatives to the null hypothesis, which is not strictly true as stated. This is shown to be true with some modification of the test. The properties of the score test have now been fully worked out, and I am glad to see that the score test has been accepted as an alternative to Wald and likelihood ratio tests. There are some differences in the properties of the three criteria referred to as the “holy trinity,” but in practice they may not lead to conflicting conclusions. The score test is invariant for transformation of parameters, a property not shared by Wald’s test.

You used to attend the meeting of a group of economists headed by Richard Stone to discuss the newly published book *Theory of Games and Economic Behavior*, by J. von Neumann and O. Morgenstern. How did you acquire this interest in economics? And did you continue with this later?

The book *Theory of Games and Economic Behavior* by von Neumann and Morgenstern was published when I was at Cambridge. Richard Stone, who later became a Nobel laureate, was a fellow of King’s College and the head of a research group in economics, and he had his office close to the museum where

I was working. I had met him on one or two occasions and had come to know of his interest in statistics. Although I had not studied economics, I bought the book by Neumann and Morgenstern and found that its contents were close to statistical decision theory, with which I was familiar. So when Stone announced a series of group meetings to discuss the contents of this book, I thought I should attend and learn more about it. Some economists at Cambridge, including Mrs. Joan Robinson, attended the meetings and contributed to the discussions. We had only a few sessions, during which we covered a few chapters of the book. It was tough going, and everybody lost interest. (I remember one famous economist making the following remark while discussing the section "two persons zero sum game." She said, "What is the role of a game in this problem? Why don't they meet and divide the available amount of money equally?" Perhaps it is more interesting to consider nonzero sum games. This was what John Nash, the Nobel Prize winner did.) I was disappointed but hoped to read the book when I was free from my statistical work at the museum. This never happened. I thought the book made an important contribution to economics and the authors would get a Nobel Prize, but this did not happen.

I wanted to develop research in game theory at the ISI. I encouraged two of my research students, T. Parthasarathy (1967) and T.E.S. Raghavan (1968), to choose game theory based on the book by Neumann and Morgenstern as their thesis topics. Both of them made valuable contributions to game theory.

I had some interest in econometrics and was instrumental in founding the Indian Econometric Society and developing its activities. I served as president and chairman of the society for a number of years. I had also organized a series of seminars on the database of the Indian Economy, to assess gaps and deficiencies in government statistics and suggest methods of utilizing the data for policy purposes. My early research on estimation and linear models is a part of econometrics literature and also, perhaps, the score test.

At Cambridge you were engaged in so many activities, including attending Fisher's classes on mathematical genetics and statistics, M.S. Bartlett's courses on stochastic processes, analyzing J.C. Trevor's anthropometric data that resulted later in a book. How did you accomplish so much in such a short time? And did these activities help you in your future endeavors? Did you think about staying a bit longer at Cambridge?

Since I had only a two-year appointment in the museum at Cambridge, I thought I should take advantage of the facilities at the university in acquiring more knowledge, especially in areas in which Indian contributions were not up to the mark. Working in Fisher's lab, I acquired some knowledge of genetics. I took a course on mathematical genetics given by Fisher. With the knowledge I acquired at Cambridge, I started giving courses on mathematical genetics to the students in Calcutta on my return to the ISI. I had also directed several theses in mathematical genetics. Two of my Ph.D. students, D.C. Rao and R. Chakraborty, are in this country and have established themselves as outstand-

ing researchers in the field of genetics. I took a course on stochastic process from Bartlett and on my return to the ISI promoted research in stochastic processes. Some of my students, V.S. Varadarajan, S.R.S. Varadhan, and K.R. Parthasarathy, have become well-known leaders of research in stochastic processes. My main aim in acquiring as much knowledge as possible during my stay in Cambridge was to expand the research activities at the ISI on my return.

My work in the museum had given me an opportunity to do research on some aspects of multivariate analysis. I extended Fisher's work on discrimination between two populations to discrimination among many populations. I developed a method of cluster analysis and the concept of canonical coordinates. These results were presented at a meeting of the Royal Statistical Society as a discussion paper that was published in the *Journal of the Royal Statistical Society* in 1948. I had also generalized ANOVA to what is now known as MANOVA and published a paper in 1948 in *Biometrika* on tests of significance in multivariate analysis.

I continued my research in the design of experiments in collaboration with K.R. Nair, who was visiting London University when I was at Cambridge. I introduced new combinatorial arrangements called "orthogonal arrays," now an essential tool in industrial experimentation. (The paper on orthogonal arrays was first submitted to *Biometrika* in 1947, but it was rejected by the editor E.S. Pearson as too mathematical and not of much practical use.) The paper was submitted to *Edinburgh Mathematical Proceedings*, where it was accepted unconditionally as an important contribution and published in 1949.

On my return to Calcutta, I started teaching combinatorial mathematics and its applications to design of experiments. As a result of this, six of my graduate students chose to write their Ph.D. theses on problems involving combinatorial mathematics and design of experiments. Two of them, U.S.R. Murthy and S.B. Rao, have made substantial contributions to combinatorics and graph theory. In recognition of my work in design of experiments and for creating an active group of research workers in combinatorial mathematics at the ISI, the Institute of Combinatorial Mathematics and Its Applications elected me as an Honorary Member with the citation, "as the world's leading expert in statistical design theory."

In addition to my theoretical work, I spent considerable time in the statistical analysis of measurements on the skeletons brought from Africa. I had a hand calculating machine to compute the variances and covariances of the multiple measurements. To compute the canonical coordinates, I had to find the eigenvalues and eigenvectors of a matrix. I did this on what was called the Mallock machine available at Cambridge University, which is an analogue machine with a device to find the eigenvalues of a matrix. I gave my report on the statistical analysis to Trevor, who, along with Mukherjee, wrote the final report. This was published as a book [6].

Before I left Cambridge, the university advertised for a lecturer's post in statistics. I had about 30 research papers published to my credit at that time. I applied for the post, but my application was not considered. The professor in

charge of foreign students at King's College sent me a note saying that foreigners are not generally appointed to teach English students. I know that the situation is quite different now, with several Indian professors working in British universities.

### **BACK TO THE ISI AND A LONG STAY: 1948-1978**

Shortly after your return to India, Professor Mahalanobis offered you a professorship and made you the head of the Research and Training School of the ISI. Was it not quite unusual in India in those days for someone to be appointed as a professor at the age of 29? What were the circumstances that led to this? Did some chance factors play a role again? What were your salaries like at the ISI in those days and afterward?

Yes, I became a professor at the ISI during my 29th year. The appointment as professor at such a young age was probably rare in India in those days, rare, perhaps, anywhere in the world. Let me tell you the circumstances that led to this event.

Just before I left Cambridge, I received an offer of a professorship at Andhra University, and on my return to India in August 1948, there was also an attractive offer from ECAFE (a UN office in Bangkok). I told the Professor about these offers and expressed my desire to work at the ISI. The Professor was pleased, and I entertained the hope that with two master's degrees and a Ph.D. from Cambridge I would get a good position at the ISI. I needed a job with a little more salary than what I was getting before, as I was planning to get married and set up a household.

As I mentioned earlier, I was first appointed in the ISI as a technical apprentice at Rs. 75 per month in November 1943, which was increased to Rs. 150 in December 1943. I was made an acting statistician in August 1944 in the grade 200-20-400-EB-25-500, (i.e., a starting salary of Rs. 200 and automatic increase of Rs. 20 per year for 10 years, and Rs. 25 thereafter crossing the efficiency bar [EB] based on good performance in the past 10 years) and had a salary of Rs. 240 in 1946 before I left for Cambridge. On my return to the ISI in August 1948, I was offered the position of Superintending Statistician in the grade (300-25-550-EB-25-800) with a starting salary of Rs. 400 and a special allowance of Rs. 100.

The ISI by that time received a grant from the central government to start a research and training school (RTS) to develop educational and training programs at the ISI and appoint faculty with academic designations like professor, assistant professor, lecturer, and so on. R.C. Bose left the ISI in 1949 to accept a position in the United States, and A. Bhattacharya accepted a professorship at the Presidency College in Calcutta. S.N. Roy and I were the only theoretical statisticians left at the ISI. The Professor told me that he was offering a professorship to S.N. Roy and an assistant professorship to me. But within a few months' time, S.N. Roy also left the ISI to accept a job in the United States.



With R.A. Fisher, ISI, Calcutta campus, 1960.

Probably at this stage, the Professor thought he should offer me a professorship as, otherwise, I might leave the ISI like Bose and Roy, although I had no intention of doing so. (J. Wishart told me that Professor Hotelling, who offered jobs to Bose and Roy, was also considering me for a position at the University of North Carolina and in that connection asked him about my skin color as there was a problem in employing dark-skinned persons at the university.)

So, I became a professor at the ISI in July 1949 in the grade (750-50-1,250) with a starting salary of Rs. 750 per month. In 1960, I reached the top of the grade as a professor and was given the designation of professor and head of the RTS in the grade (1,300-75-1,600). Finally, in 1964 the Professor offered me the position of director, RTS, in the grade (2,500-100-3,000). After the death of the Professor in 1972, I took over his designation as director-secretary till 1976, when I accepted the Jawaharlal Nehru Professorship, from which I retired in 1984. The salary remained constant at Rs. 3,000 per month from 1969 to 1984. I have given you the whole story of the positions I held and salaries I had during the period of about 40 years I worked at the ISI.

You first upgraded the one-year training course that you took in 1941 to a two-year advanced postgraduate training course. How did you get started again at the ISI in terms of your own research?

Soon after my return from Cambridge, I was busy completing the book *Statistical Methods in Biometric Research*, which I started writing while I was at Cambridge, utilizing the leisure time I had after writing my thesis. This book



was based on the work I did at Cambridge and was, perhaps, the first book on multivariate statistical methodology. Although it had theory and proofs of all propositions besides applications on real data, it was not popular with mathematical statisticians who were teaching statistics in the universities, as they did not like the discussion of practical examples along with theory. To meet their demand, I wrote the book *Linear Statistical Inference and Its Applications*, which is more mathematical and was first published in 1965 by Wiley, New York. The second revised edition came out in 1973. It was translated into Russian, Japanese, Czech, Polish, and Chinese. Special editions were brought out for sale in developing countries at discount prices. The English edition has been in demand for 35 years, and this year, Wiley is bringing out a paperback edition as a classic to make it available at a cheaper price and increase its visibility.



With Academician A.N. Kolmogorov and Professor P.C. Mahalanobis, Calcutta ISI, 1962.

Although officially I was not put in charge of the educational and training programs at the ISI, I thought of upgrading the one-year training program there. First, I made it into a two-year program and later into a three-year program. We could offer only a certificate of attendance and successful completion to the students who took this course, as the ISI did not have a charter to award degrees. The course had no rigid program of subjects to be taught with a prescribed syllabus. It all depended on the demand for statisticians in different areas of applications of statistics. In the late 1940s and 1950s I found that there was a high demand for quality control statisticians to work in industries. During those years, I put a heavy emphasis on statistical methods needed in industrial work, and most of those completing the course got employment in the industrial sector.

In 1959, Prime Minister Jawaharlal Nehru put forward a bill in the Parliament of India declaring the ISI an institute of national importance. Could you comment on the special relationship between Mahalanobis and Nehru? What role did that play in the progress of the ISI?

No other top scientist in India is known to have had quite as much connection with political parties, their leaders and activists, as Mahalanobis did. His strategy was to be friendly with all political parties, and he used to invite the leaders of all political parties to visit the ISI. I remember the visits by the top leaders of the communist, congress, and socialist parties. He gave jobs in the ISI to a number of members of the communist party, one from the socialist, and one from the congress party on the recommendation of the respective leaders. It was well known that the Professor was a good strategist and he did things with a purpose. The Indian physicist C.V. Raman once remarked that Mahalanobis knew "which side of the bread was buttered."

The Professor was particularly friendly with Nehru. He had contacts with Nehru even before Indian independence. After independence, when Nehru became the prime minister, the contacts became more frequent as Mahalanobis got involved in the development of official statistics, formulation of five-year plans, and promoting the program of inviting foreign scientists to India. Nehru was convinced of the importance of statistics, especially for planning purposes. He personally moved a bill in the Parliament in 1959 declaring the ISI an institute of national importance and authorizing it to award degrees.

I was glad that with its new degree-giving status the ISI would be able to attract very good students for graduate studies and research. The Professor insisted that we should also start an undergraduate program. I could not convince the Professor that ISI should continue as an institute for advanced studies and research and that undergraduate education should be left to the universities. I was not happy but agreed to develop a four-year undergraduate program leading to Bachelor of Statistics (B.Stat) degree in addition to a two-year graduate program leading to a master's in Statistics (M.Stat) degree. This was not an easy task, and it took almost a year of planning and discussions with various

experts to formulate the courses and to recruit the teaching staff. The first batch of students was admitted to these courses in 1960–1961. I still believe that it was not a wise decision to start an undergraduate program and that the ISI would have been better off staying with graduate studies only.

Along with the directorship of the RTS, you took on a heavy burden of guiding Ph.D. students at the ISI. You have guided many students. Debabrata Basu was your first Ph.D. student. Basu's theorem is very well known to econometricians. Many of your students are now leading authorities in various fields. Can you say a few words about your first students, such as Basu and others? What was your process of advising students? What was your typical *modus operandi*?

I started advising students for the Ph.D. degree soon after I returned from Cambridge, although the ISI did not yet have degree-giving status. We had arrangements with Calcutta University to examine the theses supervised by the ISI faculty and award doctoral degrees. This arrangement continued till the ISI was given the charter to award degrees. The first convocation of the ISI to award its own degrees was held in 1962.

My first Ph.D. student was D. Basu, whom I recruited in 1950. He had an M.A. in mathematics and did extremely well in the interview. I spent a good deal of my time with him introducing him to problems of statistical inference. He was quick in understanding and discussing the mathematical and logical aspects of statistical methods. He had a knack for constructing counterexamples to disprove some propositions as stated and proved in statistical literature. As you mentioned, he is well known for what is called Basu's theorem. This arose out of a question I put to him about the existence of a maximal (in the sense of largest) ancillary statistic. He proved the nonexistence of such a statistic and in addition proved the independence of an ancillary statistic with respect to the minimal sufficient statistic. Later he became a Bayesian and wrote papers pointing out anomalies in classical statistics. I was happy to have a brilliant and creative person as my first Ph.D. student. Others who came after Basu included A.C. Das and Des Raj, who worked in survey sampling; J. Roy in multivariate analysis; A. Matthai on quality control; R.G. Laha in characterization problems; I.M. Chakraborti in design of experiments; and V.S. Varadarajan on stochastic processes. All of them completed their work during a span of about five years and submitted their theses to Calcutta University. During this period, I had five or six students working for the Ph.D. at any given time, and Calcutta University restricted me from taking more research students to work at the same time.

When the ISI was given the charter to award its own degrees, I had the freedom to guide as many students as I could. Some of those who received the Ph.D. degree from the ISI are S.R.S. Varadhan, who worked in stochastic processes; U.S.R. Murthy, R.P. Gupta, S.B. Rao and A.R. Rao in graph theory and combinatorics; T. Parthasarathy and T.E.S. Raghavan in game theory; D.C. Rao, R. Chakraborty, K.R. Dronamraju, T.A. Davis in genetics; and I.S. Rao in di-

rectional data. At the University of Pittsburgh and Pennsylvania State University I guided Ph.D. students in a variety of areas like signal processing (D.C. Kundu, N. Kannan, and M. Bhandary), multivariate analysis (S.D. Peddada, R. Khattree, D.N. Naik, H.P. Vaidya, I. Basak, P. Basak, R. Boudreau, S. Suryawanshi, and Hyder Ali), statistical inference (N.K. Bansal, Min Deng, M. Mazumdar, and J. Pittman), and entropy measures (T. Nayak).

Till now, I have directed the research work of about 50 Ph.D. students. My strategy was to take a number of students to work in some area at the same time, expose them to current research, and assign to them different problems arising in that area. In a few cases, I had suggested problems for investigation in areas in which I had no expertise and encouraged them to read the relevant literature and discuss problems with me. I did this in order to cover different areas of current research and expand research activities at the ISI.



With Prime Minister Indira Gandhi during the inauguration of the ISI campus at Delhi, 1972.

In a recent article in the *International Statistical Review*, "Evolution of statistics in India" (1999, pp. 13–34), J.K. Ghosh and others commented on the spectacular development of statistics between 1930 and 1960. No other developing country has achieved this. Even in the United States, much of the development in statistics came later. Also in India no other discipline has achieved this remarkable development. Ghosh et al. put forward the theory of the "right man at the right time" and the extraordinary chance factor of Mahalanobis in switching his scholarly interests from physics to statistics. Do you have any thoughts on this? Did you have any problem in working with Mahalanobis?

The Professor is acknowledged all over the world as the architect of modern statistics in India. His great contributions were the establishment of the ISI, the Central Statistical Organization (CSO), the National Sample Survey (NSS), and Statistical Quality Control (SQC) units all over India, as well as the encouragement he gave to bright young people to do research and the emphasis he gave to the development of statistics as a technology for improving social welfare. He was a difficult man to work with. Some called him a "dictator." He developed too many activities at the ISI, not all of them conducive to advancement of learning and research. Personally, I had no difficulty in working with him. I admired him for "placing India not far from the center of the statistical map of the world," as Fisher put it. History will judge him as an outstanding individual who served his country well.

Although Mahalanobis is mostly known as the founder of the ISI and the other institutions that you just mentioned, he also made substantial contributions to statistical theory. Apart from  $D^2$  and innovations in sampling theory, Abraham Wald credited him for inventing sequential analysis. His fractile graphical analysis can be seen as a precursor to robust statistical procedure, and Peter Hall cites his 1946 *Sankhyā* article "Resampling methods to assess variability" and credits him as the originator of bootstrap. I do not think that there has been a proper assessment of his contributions to statistical methodologies. What are your views?

Of course, Mahalanobis's  $D^2$  is an important contribution by the Professor. His idea of pilot sampling on a small scale to yield information for designing an optimum large-scale sample survey is a precursor to sequential sampling. He introduced a method of estimating the variance of an estimator by the method of interpenetrating subsamples. In this method, the whole survey is repeated a number of times, using a fraction of the total sample size in each case, making separate estimates, and finding their variance. This method is not generally used in practice as it is expensive. The Professor also introduced econometric models for optimal planning subject to given conditions. Above all, he emphasized that "statistics must have a purpose." Data must be collected to provide the

maximum possible information with given cost to answer a specific question. This is the essence of statistics.

The Professor's contributions must be judged not in the narrow sense of theoretical papers published in journals but in the broader perspective of acquisition and use of statistics in different spheres of human activity.

Many of your ex-colleagues expressed the view that the ISI was at its best under your leadership. How did you manage to run the RTS and then the whole ISI so efficiently and also continue to carry on your own research? What was the academic atmosphere of the ISI like? Were there any problems?

I do not know how to answer this question. Judging from the bright students and colleagues I had in the 1950s and 1960s and the excellent contributions they made to different areas of statistics, ISI would probably rank well in statistics among educational institutions all over the world. Attracting good students, exposing them to an intellectual atmosphere, recognizing the work they are doing, and encouraging them to do good research is not an easy task. Somehow, there were opportunities to do all these at the ISI.

DeGroot thought the ISI was a fantastic place during the 1950s and 1960s. J.B.S. Haldane, R.R. Bahadur, and J.K. Ghosh were regular staff members. There were extremely bright students like V.S. Varadarajan, K.R. Parthasarathy, S.R.S. Varadhan, R. Ranga Rao, U.S.R. Murthy, Des Raj, R.G. Laha, and J.S. Rao to mention a few. A number of famous scientists used to visit the ISI, some for a shorter and others for a longer period. Norbert Wiener worked at the ISI for six months. Famous economists like Ragnar Frisch, Simon Kuznets, Richard Stone, J.K. Galbraith, Oscar Lange, and the famous probabilists A.N. Kolmogorov and J.L. Doob made short visits. There was no parallel to the intellectual atmosphere that existed at the ISI during the 1950s and 1960s.

However, there was a problem that made me unhappy. When I was working at the University of Illinois, Urbana-Champaign, in 1953–1954 as a visiting research professor of mathematical statistics, I had access to Illiac, the first digital computer in the United States. I took a course on programming using machine language and started using the computer to do computations. The University of Illinois gave me two students to work with to develop computer programs for statistical methods. I thought I would have an opportunity to use computers on my return to India to do research involving heavy computations, which was not possible with hand-driven desk computers. The ISI had acquired a digital computer, but the workers' union at the ISI was against the use of computers. They prevented me from using the computer. It was difficult to convince them, and I think we lost the opportunity to develop computer-intensive statistical techniques in India well before others did. The clock was set back by 20 years. This was a big disappointment for me. I sent a letter to Professor Mahalanobis resigning from the ISI and stopped going to office. The Professor and other administrators of the ISI persuaded me to withdraw my resignation

to avoid a slur on the workers and maintain the reputation of the institute. I agreed to continue to work in the best interest of the institute.

During this period you also won many awards and fellowships including the Bhatnagar Award in 1963, being made a Fellow of the Royal Society (FRS) in 1967, and one of the Indian government's highest civilian awards, the Padmabhushan, in 1968. Any comments on these?

Getting awards is not easy. Good work is not sufficient. You must have an influential person who is kindly disposed to you to sponsor you for any award. For the Bhatnagar Award of 1963, Dr. Bhaba, director of the Tata Institute for Fundamental Research (TIFR) in Bombay, and Professor Mahalanobis were on the selection committee. They decided to split the award between Dr. Chandrasekaran, who worked in TIFR, and me. They probably thought that the awards were meant to enhance the image of the institutions they were heading. The Award of Padmabhushan in 1968 was a consequence of my being made a Fellow of the Royal Society in the previous year. The Professor nominated me for the Royal Society as Bhaba nominated Chandrasekaran. I was fortunate to be made a Fellow of the Royal Society.



With Jerzy Neyman, Mrs. Rao, and R.C. Bose, 1974. The main building of the ISI Calcutta campus is in the background.

Talking of awards, I must mention an amusing incident. Three years ago, the government of India issued a notice published in all newspapers in India announcing the institution of two cash awards, one "in memory of P.V. Sukhatme" to a statistician and another "in memory of C.R. Rao" to a young statistician below the age of 45 for outstanding work in statistics to be given every alternative year. These awards were meant to be a tribute paid to P.V. Sukhatme, who died a few years earlier, and myself presumed to be dead. (Perhaps my absence from India after retiring from the ISI gave the impression to government officials that I was not living—perhaps an instance of "out of sight, out of mind.") When it was pointed out that I am alive and kicking, the government issued an amended notification that the second award is "in honor of C.R. Rao." I felt happy that my contribution to the development of statistics in India received some recognition by the government. I am reminded of the story of a politician who asked his secretary to spread the news that he died in order to know what the newspapers would report about him after his death.

From your list of publications, it is safe to infer that your productivity continued to increase over time. How did you achieve that?

When I started working at the ISI, there were only a few people doing research. I had no one to discuss and collaborate with. I was left to myself, and most of my early publications had no coauthor. Later, when I was guiding the research of Ph.D. students, I followed the policy of not associating my name with papers arising out of their theses, even when I had a large input. In fact, I have no joint papers with any of my Ph.D. students on problem arising out of their theses. Much later, when there were a sufficient numbers of senior researchers at the ISI, I started writing papers jointly with others. When I moved to the United States, I had grants for inviting senior people to work with me on problems of mutual interest. The number of papers one writes depends also on the number of collaborators. This explains why I have more published papers in later years. I do not know whether you as an economist would call this an increase in productivity. Many of the results that bear my name were derived in my early papers when I was working by myself.

Even with your busy schedule as the head of RTS and later as the director of the ISI, you continued to travel, attending various international conferences and visiting universities abroad. Would you comment on your travel activities? How did these benefit you? As you just mentioned, during 1953–1954 you were a visiting research professor of mathematical statistics at the University of Illinois, Urbana-Champaign. What recollections do you have of that visit?

The latest count of the number of international conferences I attended is close to 200. The first one was the Colloquium on Probability and Statistics held in Lyon, France, in 1948. There, I met Doob, Frechet, and a few other well-known probabilists. I also met LeCam, who was still a student planning to go



to the United States for higher studies. I have traveled to almost every country in the world to attend conferences.

While I was working in India, every 10 years or so, I took leave to accept visiting appointments in the United States. The first one was in 1953–1954 at the University of Illinois, Urbana-Champaign, and the University of California, Berkeley. The second was in 1962–1963 at Johns Hopkins and Stanford, and the third was in 1972 at Indiana University, Bloomington. The fourth was in 1978–1979 at the Universities of Pittsburgh and Ohio State. I retired from the ISI in 1980 at the mandatory retirement age of 60 and started working in the United States. I have just retired from Penn State after 40 years of service at the ISI and 20 years at the Universities of Pittsburgh and Penn State.

My periodic visits abroad during my career in India were academically rewarding. I had the opportunity to meet personally most of the well-known American probabilists and statisticians.

At the University of Illinois, I gave a course on multivariate analysis. Besides regular students, some staff members from the psychology department, who were working on factor analysis, attended the course. I became interested in factor analysis while talking to them and wrote a paper on what I called “canonical factor analysis,” which was published in *Psychometrika* [85]. I remember using the Illiac to do some computations for the paper. I had a student named Gene Golub who is now a member of the National Academy of Engineering and National Academy of Sciences.

#### IN THE UNITED STATES: 1979–

After 40 years of service at the ISI perhaps you were contemplating a quiet, retired life. But that was not to be. In the fall of 1979 you found yourself settling down at the University of Pittsburgh in what was an almost a brand new career in a new environment. Could you recollect the events leading to your departure from India to the United States?

After the death of the Professor in 1972, I took over his position as the secretary and director of the ISI. This gave me an opportunity to change the constitution of the ISI to simplify administrative procedures and give more autonomy to the different research divisions. The title of the head of the ISI was changed from secretary and director just to director, with a five-year term in office with a provision for reappointment. The new constitution was adopted in 1976 by the administrative council of the ISI. I expressed a desire not to continue as the head and to accept the Jawaharlal Nehru Professorship (JNP) created by the Professor before his death for me to take up when I retire from or give up headship of the ISI. It was very thoughtful of the Professor to create the JNP as an independent unit within the ISI providing for a salary equivalent to that of the director and a separate budget for supporting administrative and research staff. The council agreed to appoint a new director and offer me the JNP.

Two things happened. The chairman of ISI, who presides over council meetings, downgraded the provisions of the JNP and remarked that my services were

not essential for the institute. However, I continued to work as the JNP but felt somewhat uncomfortable adjusting myself to my new position. I thought that I should take leave for a year and go abroad to work in a different environment. In 1978, I accepted a three-month appointment at the University of Pittsburgh as Mellon Professor, followed by a nine-month appointment at Ohio State University as a visiting professor.

I found myself welcome at both places. The Ohio State University awarded me an honorary D.Sc. and offered me a tenured professorship in the statistics department. The dean of the University of Pittsburgh offered me a university professorship. I accepted the latter, as my son was studying at the University of Pittsburgh, and as a staff member, I would have the additional benefit of not paying tuition for my son. I took additional leave from the ISI and joined the University of Pittsburgh in the fall of 1979.

When I took a one-year leave from the ISI in 1978, it was my intention to return and continue as the Jawaharlal Nehru Professor. But I did not expect that I would have attractive offers in the United States at my age—nearing 60. I was also aware that I would be facing some psychological problems in working at ISI after stepping down from the headship. This happens in India, where people respect authority more than individual accomplishments.

My five-year term as JNP expired in 1985. I was then made a National Professor by the government of India, a position that I accepted without pay. (The number of National Professors in all areas is limited to 12 at any one time.) In 1992 I gave up the National Professorship when I decided to become a U.S. citizen.

Looking back over your academic career, how would you characterize the main differences between the United States and India? Certainly you became even more productive (if I may use the term again) after moving to the United States.

Of course, there are some differences between working in India and the United States. In the United States, I had no administrative responsibilities and could devote all of my time to research. Furthermore, I had the opportunity to interact with statisticians and mathematicians working in areas close to those of my interest. In India, we do not have enough people engaged in active research in the universities or even in the research institutions. It is difficult to find a group of people working in collaboration to solve problems. At the University of Pittsburgh, I used to give research seminars, which were attended by faculty members in statistics and mathematics. Some problems I mentioned in the seminars were picked by mathematicians, which resulted in several joint papers with them. In addition, I had grants to support visitors from abroad and the United States, and collaboration with visitors resulted in several joint papers.

Some important results that came out of my collaboration with the mathematicians at the University of Pittsburgh are the Lau-Rao theorem [224, 242], obtained jointly with Ka-Sing Lau, a topologist, which plays an important role in solving characterization problems, and Burbea–Rao divergence measures [226,

228, 229], obtained jointly with Jacob Burbea, a differential geometrician, which are useful in constructing specification test criteria as alternatives to chi-squared tests.

In India, you have to work more or less in isolation with practically no opportunity to discuss problems of your interest with others. We are not able to do first-rate research in India because of this lack of “critical mass” in terms of researchers and resources in any research or educational organization. This probably accounts for having less productivity when you are working in India.

There were some periods during my long association with the ISI when I did not have colleagues or bright students with whom to discuss problems. In addition, I had heavy administrative responsibilities, especially after the death of Professor Mahalanobis. As a result, I was not able to pursue some new lines of research that I hoped to develop.

Before leaving India, you organized the 41st session of the International Statistical Institute in New Delhi. Was it difficult to organize such a big conference? I myself have very fond memories of this meeting. Although I was in the Economics Division of ISI-Delhi Campus, during this meeting I met many world-famous statisticians. It had a tremendous impact on me. I am sure many of my fellow students had a similar experience. I cannot recall any other major statistical conferences held in India since then.

Although it was not my responsibility to organize the 41st Session of the International Statistical Institute, I thought I should help given my experience in holding conferences and the international contacts I had, in order to make it a success. It was not easy to seek the cooperation of all concerned, to get things done on time, and to organize the scientific and cultural programs. I had to work hard with only one assistant to help me. There were numerous problems to be solved, such as getting visas for delegates from South Africa, issuing commemorative stamps, bringing out the proceedings of the conference, and observing protocol in dealing with the ministers and secretaries of government departments. I was relieved when the conference ended without a hitch.

One incident might be of interest to you. At the opening of the conference, I talked about brain drain and suggested what the government could do to control it. Morarji Desai, the then prime minister who inaugurated the Conference, said that he was not worried about our scientists leaving the country and going abroad to take up jobs. He expected scientists to be patriotic.

What did you miss most in the United States that you had at the ISI? Particularly at the Calcutta ISI campus, it was a common sight to see you walking around the beautiful large ponds accompanied by your students and colleagues intently discussing research problems, what the students called “pondering”! Have you been able to continue that practice in some way here?

I had a nice time in Calcutta on the campus of the ISI. I was responsible for building residential quarters for staff members and hostels for students on the campus. This gave the staff and students opportunities to meet and participate in intellectual activities, socialize, and play outdoor games.

I usually played badminton with staff and students in the evenings. I was a fairly good player. A colleague of mine, A. Maitra, and I were declared the best badminton team in the ISI. When students wanted to see me, and I had no time in the office to meet them, I would ask them to join me in the evening in walking round the ponds on the ISI campus and discuss their problems. In the United States, I go for a walk every day with my wife. We try to enjoy nature and not to solve any domestic problems.

Did you keep in touch with the ISI, and India in general, during these years in the United States? Do you have any official relationship with the ISI now?

I try to keep in touch with the ISI. I go to India almost every year and visit two or three branches of the ISI. I do not have any official relationship with the ISI except as chairman of the board of directors of the International Statistical Educational Centre (ISEC). The ISEC was started at the ISI in early 1950s in cooperation with the International Statistical Institute and UNESCO. This is a one-year program for the officers deputed from foreign countries, mostly from Southeast Asia and Africa. I was made the director of ISEC in 1972 after the death of the Professor, who was the first director.

When I left the University of Pittsburgh to accept the Eberly chair in statistics at Penn State, I was offered adjunct professorship at the University of Pittsburgh, and I am continuing in this position in addition to Emeritus Eberly Professorship at Penn State. I expected to have some connection with the ISI in some capacity. But unfortunately this did not happen.

What were the circumstances for your move (at age 67) to Pennsylvania State University in 1988 after working for eight years at the University of Pittsburgh?

P.R. Krishnaiah, whom I had known for a long time, was the head of the statistics group in the mathematics and statistics department at the University of Pittsburgh. He was responsible for sponsoring my visit to Pittsburgh. Unfortunately, he died of cancer in 1987. I suggested to the dean the name of an outstanding statistician to fill the post of Krishnaiah—but, unhappily, this drew a negative response. At that time I received an offer from Penn State of a new chair in statistics established with a donation from the Eberly family. I accepted the offer, although the University of Pittsburgh tried to match my salary and also offered to create a professorship in my name when I retire.

When I joined the University of Pittsburgh in 1979, the statistics section in the mathematics and statistics department was in bad shape. Within a period of two years, Krishnaiah and I developed graduate programs in statistics leading to master's and Ph.D. degrees. We also established, with the support of grants

from the navy and air force, a Center for Multivariate Analysis, the only one in the world with a primary focus on research in multivariate analysis, and initiated several research programs in collaboration with visiting scientists from the United States and abroad. As a result, Pittsburgh's Department of Mathematics and Statistics was rated the "most improved" of 63 statistics doctoral programs in the United States in a survey conducted in April 1981 by a committee of the Conference Board of Associated Research Councils. The University of Pittsburgh acknowledged this as due to the recent appointment of two "outstanding statisticians," P.R. Krishnaiah and myself.

Did your presence at Penn State have a similar effect?

The Statistics Department at Penn State received a good rating in a review of the universities in terms of published research work by Christian Genest, the editor of the *Canadian Journal of Statistics*. This happened after I joined Penn State, but I do not know whether my presence had anything to do with it. (I learned from Genest that according to his statistics, I was the most prolific writer among the staff of the statistics department during the 1990s.)

Let me now ask you a question that comes under the purview of counterfactual history. What would have happened had you moved to the United States much sooner, as many of the early recruits of Mahalanobis did? As you mentioned earlier, Professor H. Hotelling wanted to recruit you for a position at the University of North Carolina. Do you have any regrets?

I could have moved to the United States much earlier as I had some offers. But I declined because I thought working at the ISI would enable me to develop statistical research in India. It meant some sacrifice on my part in many ways, but I had the satisfaction of promoting statistical education and research in India at a level comparable to that in any other country.

If I had moved to the United States at a younger age, I might have benefited personally. But I enjoyed my stay in India in spite of the difficult conditions under which I had to work. I thought that I earned a great tribute when an old colleague of mine remarked at the conference held in Calcutta to celebrate my 80th birthday that "Rao stood firm with Professor Mahalanobis when others left him and helped in building the ISI." As I mentioned earlier, administrative responsibilities and other problems in India prevented me from pursuing some new lines of research that I hoped to develop.

Also during your tenure in India (1948-1979), did you ever feel cut off or left out from the centers of statistical activities in the United Kingdom or the United States?

Not very much, mainly because of my periodical visits to the United Kingdom and United States. I had invitations to attend conferences and give invited talks and to visit statistics departments of universities to give seminars. I felt a little isolated but not left out.



Receiving the National Medal of Science from President George W. Bush at the White House on June 12, 2002.

I was surprised at the unsolicited attractive job offers I received when I visited the United States in 1979 shortly before retirement from the ISI—a sure sign of being not left out. I was awarded honorary doctorate degrees by three universities in the United States, the Wilks Medal by the American Statistical Association, and was elected as a member of the National Academy of Science and the American Academy of Arts and Science. Above all, I received the most prestigious award given to an American scientist, the president's National Medal of Science. I did not expect these honors, being a foreigner who spent a major part of his working life in India. This can happen only in a country like the United States, which attaches a great value to scholarship and individual achievements.

### **ECONOMETRICS**

You always had a soft spot for econometrics. One of your first research papers, as we discussed earlier, originated in a problem of Ragnar Frisch. You were one of the founders of the Indian Econometric Society in 1960. You guided the activities of the society as its president from 1971 to 1976 and continued your active association as chairman of the

society for many years. We have so many brilliant Indian statisticians, what J. Neyman called "a thousand bright stars in the statistical sky." However, the same cannot be said in the field of econometrics. Why is that?

Statistics flourished in India mainly because of the early establishment of the ISI and the role it played in promoting statistical education and research. R.A. Fisher said in one of his lectures at the ISI some years ago that "the number of Indian statisticians is more than the number of statisticians in all the other countries put together," and he attributed this phenomenon to the existence of the ISI. Perhaps there has been no such comparable organization in economics in India to promote education and research in economics. The Delhi School of Economics founded by the well-known economist V.K.R.V. Rao had the potential to promote fundamental research. The universities in India concentrate more on teaching from textbooks and are unable to produce good research. Most of the research publications of good quality come from specialized institutes like the TIFR, ISI, and so on. Perhaps there is a need for the establishment of an advanced institute in economics to promote research of good quality.

Econometrics has a strong statistical basis and is, in fact, taught as a special subject in graduate programs in statistics. I tried to promote research in econometrics in India through the Indian Econometric Society, by organizing conferences and seminars.

At the Delhi branch of the ISI, I tried to create a strong center for research in mathematical economics and econometrics. Several well-known Indian economists were employed and given opportunities to develop educational and research programs in economics. Due to some reason or other, the project did not take off as expected.

You edited the *Handbook of Statistics in Econometrics* with G.S. Maddala and H.D. Vinod and then another with Maddala on *Statistical Methods in Finance*. What prompted these collaborations? Did these efforts facilitate some interaction between statistics and econometrics? Speaking of G.S., he claimed to be "almost" your student (see the interview by Kajal Lahiri in *Econometric Theory*, 1999, 15, 764).

I decided to produce a volume on econometrics in the series *Handbook of Statistics* because of the use of statistical methods in econometrics. G.S. Maddala was quite knowledgeable in this area and had done a good job in editing the volume. I believe this volume has been well received by economists and statisticians, as has the volume on financial statistics that he edited. He was not my student. I offered him a fellowship at the ISI after he got his master's in statistics, but he did not accept.

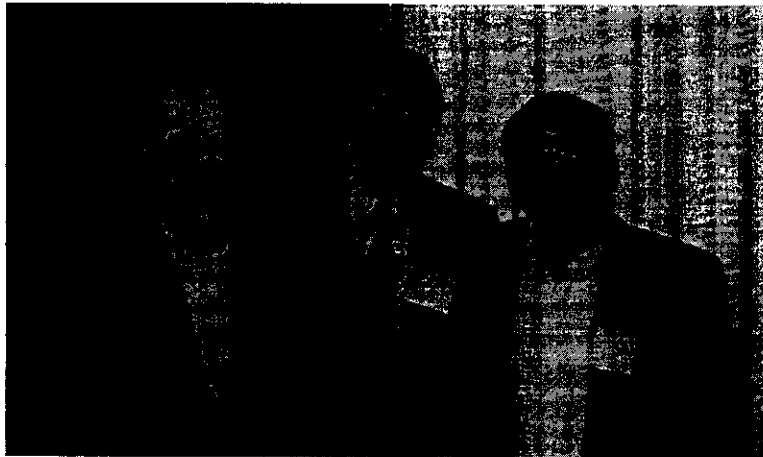
I met Maddala during my visit to Stanford in 1963 but did not interact with him. We saw each other very often after my arrival in the United States. I had

great admiration for his vast knowledge of not only economics but also topics of general scientific interest. He was kind enough to edit a festschrift on my 70th birthday.

Much of your work such as the Cramér–Rao efficiency bound, the Rao–Blackwellization, the score test, MINQUE theory, the  $F$ -test, quadratic entropy, your distance measure, and  $g$ -inverse has had profound impacts on the theory and practice of econometrics. Would you make some comments on these usages?

My results on estimation and hypothesis testing can be used in data analysis in any field. (The Cramér–Rao lower bound is often quoted in papers on electrical engineering.) Perhaps they have greater relevance in economics, especially in the discussion of econometric models. The estimation of parameters in a specified model is a statistical problem, as is the choice between alternative econometric models based on given data. There are some unsolved problems in the area, which may require collaboration between statisticians and economists.

However, I was surprised to see the book by Uwe Jensen, *Herleitung, Berechnung, und Okonomische Anwendung von Rao-Distanzen*, where the author develops tests for hypotheses arising in economics, based on the geodesic distance function between probability distributions introduced in my 1945 paper on estimation [33].



With Zhijie Xiao and Anil Bera during the Eighth Lukacs Symposium, *Statistics for the 21st Century*, held at Bowling Green State University, April 24–26, 1998.



In an interview with Morris DeGroot you expressed some concerns on the applications of statistics in economics (*Statistical Science* [1987, 67]): "I don't think we are very successful with statistical methods in psychology or even in economics. Possibly what is wrong with the economists is that they are not trying to refine their measurements or trying to measure new variables which cause economic changes." Has your view changed given the recent advances in econometrics?

What I said in DeGroot's interview about the use of statistics by economists needs some elaboration. When we construct a model to predict response variables in terms of explanatory variables, an implicit assumption is made that the explanatory variables are observable without error. In the hard sciences measurements can be made very accurately. It would be of interest to investigate how accurate economic measurements are and what is the effect of measurement errors in prediction. Such studies are available for simple models, but not much is known in the case of complicated models like we have in economics.

You may have heard of the familiar sayings "The economy's slide has one familiar feature: few if, any, economists predicted it." "The larger paradox is that economic forecasts are least reliable when they are most needed." This, I believe, is due to the omission of some variables in the prediction model, such as technological changes, political, and others, which can introduce "shocks" into the system and introduce errors in forecasts. Economists also talk of "consensus forecast," the validity of which has not been properly explored.

The ISI was at the forefront of building five-year plan models for India under the leadership of Professor Mahalanobis. In the early 1950s there was intense activity at the ISI Calcutta campus. Mahalanobis invited many prominent economists to the ISI during that time, including Ragnar Frisch, John Kenneth Galbraith, Simon Kuznets, Oskar Lange, and Jan Tinbergen. Were you involved in this project?

I was not involved in the work done by Mahalanobis and the ISI on planning models. I used to meet all the famous economists invited by Mahalanobis. Some of them participated in the work of the planning division of the institute, but I am not sure whether they have made any substantial contribution to the formulation of our five-year economic plans. Perhaps the Professor, who was in constant touch with them during their stay at the ISI, got some ideas from them in formulating the five-year plans.

## GENERAL

Let us now talk about some general topics. Family comes first. From Cambridge you returned to India in August 1948. Then you got married on September 9, a day before your 28th birthday. At the time of your marriage you were also finishing the book *Advanced Statistical Methods*

*in Biometric Research*. How did you manage to do that? As you have acknowledged on several occasions, the greatest support you received in your six decades of research work is from Mrs. Rao, and all other support in terms of opportunities and research facilities and grants was secondary. It would be nice if you could say a few words about your family life.

I started writing the book *Advanced Statistical Methods in Biometric Research* when I was in Cambridge, and I wanted to complete it on my return to India. Due to pressure from my family I got married within a month of my arrival in India. This created some domestic problems, as I was dividing my time between writing the book and keeping company with my new wife. But soon she realized the importance of the work I was doing and adjusted herself to a life she was not used to before marriage. She has two master's degrees, one in history from the Benaras Hindu University in India and another in psychology from the University of Illinois, Urbana-Champaign. When we were in Calcutta, she worked for some years as a high school teacher and for a number of years as a lecturer in the Birla Institute of Home Science affiliated with the Jadavpur University. We have several common interests, and we have been now happily married for over 53 years, "not without those minor quarrels which Providence sends to enhance the pleasures of life" (as Oliver Goldsmith said). I owe her for the environment she provided at home to pursue my research.

We have a daughter, Tejaswini, and a son, Veerendra. Tejaswini has a Ph.D. in nutrition and is now an associate professor at SUNY, Buffalo. She is also an accomplished Indian classical dancer and runs a dance school in Buffalo. Because of my own interest in Indian classical dance (I was president of the Kuchipudi Dance Academy, one of the major Indian classical dance styles, in Delhi for a number of years), we put our daughter in a dance school at the age of eight. Veerendra is an electrical engineer and is a vice president of a computer consulting company.

As your past students will attest, you are a superbly accomplished teacher. Many of them say how you made difficult mathematical concepts so simple, using good humor and interesting anecdotes. Professor Nikhilesh Bhattacharya of the Economic Research Unit, ISI-Calcutta, possibly one of your first students in the 1950s, recently recalled in a meeting that you used to give "tailor's measurements" as an example of a "vector." What a nice example! Here is a quote from a recent Penn State student in your multivariate analysis class: "Watching Professor Rao lecture is like watching a skilled artist at work, with every statistical function and procedure at his command." Have you yourself had some inspiring teachers? What is your teaching philosophy?

I like teaching. I remember when I was in school, my classmates used to come to my house to learn what they had not followed in the class. In my teaching, I try to place greater emphasis on ideas and concepts rather than stating propo-

sitions and proving them. Often, I would develop the main arguments in a proof and ask the students to fill in the details. I learned this technique from one of my teachers, V. Ramaswami, when I was a mathematics student at Andhra University.

I generally get high ratings from my students for my teaching. Many of them have written to me in appreciation of my ability to put across difficult concepts in a simple and understandable way. I was once deeply moved when my students gave me a standing ovation at the end of the last class in the multivariate course I taught at Penn State.

You have received numerous awards, 27 honorary doctoral degrees from universities in 16 countries around the world, and honorary fellowships from the most prestigious societies. In January 2001 the government of India bestowed upon you Padma Vibhushan, the second highest civilian award, for your outstanding contribution to science, engineering, and statistics. On top of all these, most recently you received the President's National Medal of Science, the highest science award given in the United States. How do you feel about these awards?

I am reminded of what Fisher said about the recognition due to scientists for their achievements.

A ballet dancer gets her ovation on the spot, while she is still warm from her efforts. A wit gets his laugh across the table; but a scientist must expect to wait about five years for *his* laugh. Recognition in science, to the man who has something to give, is, I should guess, more just and more certain than in most occupations but it does take time. And when it comes it will probably come from abroad.

The first award I received came from abroad. It was Fellowship of the Royal Society, UK. Then the rest followed, the government civilian award and all the honorary doctorates.

Recently I came across the comment that Ramakrishna needed Vivekananda to send his religious message to the masses and reform Indian society. And similarly Mahalanobis had you to fulfill his dream and vision of bringing India to the forefront of statistical research and training. How do you feel about this characterization?

Of course, Professor Mahalanobis had the vision to develop the new discipline of statistics in India and the mission to propagate statistical knowledge for use in enhancing individual and institutional efforts. These objectives were clearly stated in the constitution of the institute he founded. It is true that the Professor depended on those who worked at the institute to fulfill his dreams. It is for posterity to judge the course of events that took place during the Professor's lifetime and after his death.

I tried to carry out the Professor's mission in spite of the many difficulties I had to face. My best achievement is not my research work. It is the environment and opportunities I created for my students for their intellectual develop-

ment, to formulate new problems and seek their solutions. I am proud that several of my students, after their Ph.D., continued their pursuit of knowledge and have become leaders of research in their areas of specialization.

Did any of the ISI workers feel any "bitterness" toward Professor Mahalanobis? You worked with him for more than 30 years very smoothly. You also maintained a very good relationship with Neyman in spite of being a direct student of R.A. Fisher, with whom he had a bitter relationship. What is your secret? Also can you elaborate on the mutual relationships among Fisher, Egon Pearson, and J. Neyman? You are now the only bridge to that vanished world. Many people will be keenly interested in your recollections and observations.

My relationship with the Professor was unique. He took me into his confidence and freely discussed the problems of the institute. We had differences of opinion from time to time on administrative matters. But this did not affect my work at the ISI. I do not think anyone at the ISI had bitterness toward the Professor, but some did not like the autocratic way he ran the institute. There were some cases when he was not kind to those who differed with him and made them feel that he was the boss.

There were bitter debates between Fisher and Neyman on testing of hypotheses and analysis of variance tests in design of experiments. In retrospect, we can see that there are lacuna in the theories propounded both by Fisher and Neyman. Perhaps much was due to conflict of personalities and Fisher's intolerance to criticism of his work. Neyman was by disposition a very kind man. Although I criticized his work, he used to invite me to his home whenever I visited Berkeley. Neyman was the first to notice my work on the bound to variance of an unbiased estimate and gave it the name Cramér–Rao inequality. When I was visiting the University of Illinois in 1953–54, Neyman invited me to spend the summer in Berkeley and teach a course. When my wife and I reached Berkeley by train, he personally came to receive us and treated us kindly. He was also thinking of giving me a job in Berkeley, but for some reason this did not happen.

Only recently, the Bengali media has been discussing many soft sides of Mahalanobis. Nobel laureate Rabindranath Tagore always admired Mahalanobis. In fact, their admiration was mutual. Did Mahalanobis ever discuss with you his love for Tagore's writings and music?

The Professor used to refer to Tagore frequently, and he would tell me Tagore's views on the political and social issues of India. When I had some problems with my colleagues, he tried to comfort me by quoting from Tagore, of which the English translation is as follows: "Oh unfortunate mother. You have kept 70 million of your children as Bengalis. But you have not made them men."

Your *Linear Statistical Inference and Its Application* (2nd edition, 1973) is still regarded as a bible of statistics. Did you ever think about bringing out a third edition? *Statistics and Truth: Putting Chance to Work* (1989) is a delightful little book. It has already been translated into five other languages. What prompted you to write this that has become so popular?

I am surprised that my book *Linear Statistical Inference* has been in the market for over 35 years, and as I have mentioned earlier, Wiley is bringing out a paperback edition to increase its visibility and sell it at a lower cost. *Statistics and Truth* was originally published as a series of lectures I delivered in India during Ramanujan's Birth Centenary at the invitation of the Indian Council of Scientific and Industrial Research. It was well received, and a suggestion was made for reprinting it. I thought I should present the material of the lectures in book form, adding some new material. The revised book was published as a second edition by World Scientific in Singapore and has been translated into Chinese, German, Japanese, Polish, and Spanish.

I remember reading your book *Computers and the Future of Human Society* (1970) many years back. There, you made some prophetic statements about the advancement of computer technology. Any comments about that?

*Computers and the Future of Human Society* was also a collection of lectures on modern computers I gave at Andhra University. That was a long time ago, when universities and institutions in India were beginning to acquire computers for research. Since then there has been a tremendous increase in computing power, and we are all aware what effect computers are having on our research and daily activities.

You have worked on a vast number of topics covering almost every area of statistics. Is there any topic that you wanted to cover but could not? Any particular advice to young researchers?

One never knows. New uses of statistics are discovered from time to time. There are wide-open fields for young research workers to sow the seeds and reap the harvest.

Mathematician G.H. Hardy noted in his 1940 memoir, *A Mathematician's Apology*, "No mathematician should ever allow himself to forget that mathematics, more than any other art or science, is a young man's game." Certainly you are a counterexample of this. You are now 81 and retired only recently. You are about to travel to Australia to collect yet another honorary doctorate. So what does the future hold for you?

I am trying to continue my research more to keep my mind alert and prevent deterioration of brain cells. I have read *A Mathematician's Apology*. It is true that all bright ideas in mathematics came from mathematicians when they were

young. Mathematics depends on abstract and uninhibited reasoning, which decreases with age. But statistics is different. It depends on practical wisdom, which old people are credited with.

Let me now ask you the unanswerable question that I think everyone would like me to ask. What is the source of the many fundamental ideas you have advanced?

When I was young, there was not much literature on statistics. It was a young discipline, and anyone could acquire most, if not all, of the available knowledge. It was wide open for anyone to make new contributions. Most of the statisticians of my generation had wider interests and contributed to several areas of statistics. But with the rapid development of statistics, there is now enough material for students to specialize in a narrow area of statistics, as in mathematics. Their research remains confined to such areas of specialization.

The 20th century has been wonderful for statistics. During this century, statistics has developed from its infancy into a mature science. What direction would you like to see the statistics profession take in future?

Next week you are going to give the 2001 C.G. Khatri Memorial Lecture here at Penn State. The title of your talk is "Has Statistics a Future? If So, In What Form?" Can you give a very short preview of your talk for the benefit of the *ET* readers?

It is true that statistics found a place as a method of drawing inferences under uncertainty in all fields of human activity soon after the mathematical founda-



Group photograph during the International Conference on *Statistics: Reflection on the Past and Visions for the Future* in honor of C.R. Rao, on the occasion of his 80th birthday, March 16–19, 2000, University of Texas, San Antonio.

tions were laid by K. Pearson, R.A. Fisher, J. Neyman, and A. Wald in the first half of the last century. There are specialized textbooks on theoretical statistics and on applications in business, industry, medicine, law, and even arts and literature. However, there are anomalies and controversies in the use of statistical methods and the interpretation of results within the framework laid down by Fisher, Neyman, and Wald. The choice of test statistics, the use of  $p$ -values in tests of significance, and quantification of uncertainty in decision making as frequency of wrong decisions in repeated sampling of a fixed size are still being debated. Also, there are Bayesian statisticians who advocate the use of prior information in statistical analysis, without laying down the rules for the choice of that information. The situation is somewhat chaotic, and in a recent study, it was found that different statisticians working on the same data often come to different conclusions.

Much of the current statistical methodology is model based, without any guidelines for the choice of the underlying stochastic model for data. The methodology developed was geared to the analysis of small data sets (samples). However, with modern technology and available resources, it is now possible to generate large data sets in any investigation. This raises new problems in computing and the possibility of extracting information from data without using a stochastic model. A new methodology is being forged for this purpose under the name of "data mining" or "computational statistics." In my Khatri Memorial Lecture, I propose to survey the development of statistics in the last century, highlight the controversies in the use of statistical methods, and discuss some of the modern trends.

Do you have any general advice to give to prospective graduate students of economics and statistics?

I have no particular advice to graduate students in economics and statistics. They have to study whatever the universities can offer them to earn a degree. They may find the knowledge they have acquired and the skills they learned in the universities are outdated by the time they acquire a degree and not sufficient to solve the problems in the professions they enter. They may have to update their knowledge by reading journals and attending short courses in specific areas. Above all, they have to use their new knowledge and forge appropriate tools with special reference to problems they are required to solve.

Apart from statistics you have a plethora of other interests, such as gardening, photography, cooking, and Indian classical dance. Tell us a bit about these. How did you find time to develop these hobbies with your busy schedule?

I have been interested in cultural activities since I was quite young. I had the opportunity to attend musical and classical dance concerts when I was working in Calcutta. I accepted the presidentship of the Andhra Association in Calcutta and organized several cultural events. I enrolled my daughter in a dance school

when she was eight years old and gave her the opportunity to attain professional level in two Indian classical dance styles. When I was in Delhi, I accepted presidentship of the Kuchipudi Dance Academy.

I have enjoyed gardening since I was a boy and have kept my interest throughout my life.

I developed an interest in photography after I got married and found a suitable subject in my wife. I kept my interest in photography, although it is an expensive hobby. Some of the photos I have taken have appeared in newspapers and photographic journals.

I love outdoor sports. I used to play soccer and badminton, and I encourage my students to play games, as I believe that the development of the body and the brain should go together.

When I was in Cambridge, I learned how to cook Indian food out of necessity. My son is a good cook, and I believe he inherited his culinary talents from me.

You have asked how I found time to develop so many hobbies. The answer is, perhaps, the absence of TV when I was growing up.

It is said that there is a woman behind every successful man. My wife, who has two master's degrees, one in history from an Indian university and another in psychology from the University of Illinois at Urbana-Champaign, provided the intellectual and congenial atmosphere needed to pursue my creative activities.

Dr. Rao, it has been wonderful to meet and talk with you. Thank you so much for giving me this interview. I wish you many more years of happy and productive life.

Thank you.

#### PUBLICATIONS OF C.R. RAO

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