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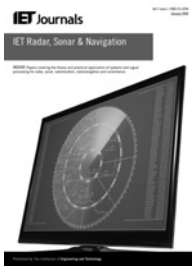

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Conjugate directions based order recursive implementation of post-Doppler adaptive target detectors

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Abstract: An implementation for the post-Doppler adaptive target detectors enabling an efficient change of the subspace dimension is described. The proposed implementation uses the order recursive structure of the conjugate directions method and does not present any additional computational burden on the processor. The implementation can be particularly useful for the adaptive detectors with an indeterminate number of auxiliary vectors for the clutter covariance matrix estimation. Through the proposed method, the subspace dimension can be easily increased or decreased according to the multiplicity of the auxiliary vectors at no loss of computational efficiency.

1 Introduction

A radar system is to reliably detect targets under the interference of noise, clutter and jamming. The detection problem becomes further challenging when the strength of the interfering signal and its statistical properties are not known a priori. For such problems, the interference has to be estimated during the operation and the detector has to operate with the imperfect characterisation of the interfering signal. Such target detectors are called adaptive target detectors [1, 2].

Following the seminal work of Kelly [2], a number of adaptive radar detection techniques have been proposed [3–6]. Especially for the applications with non-stationary clutter, as in the airborne radar applications, the usage of adaptive detectors havs found much success [7, 8]. Different from the conventional detection techniques, adaptive detectors estimate the parameters of the interfering signal using a set of auxiliary data. The auxiliary data are assumed to have the same statistical properties with the cell under test and are generated by probing the cells in the neighbourhood of the cell of interest. The auxiliary data are processed to estimate the covariance matrix of the interference signal and the goal is to utilise the estimated matrix to cancel the clutter in the cell of interest [9–11].

In this paper we present an adaptive target detector implementation for the post-Doppler operation. The suggested implementation does not have any additional computational burden over the conventional technique, but it allows an easy and cost-efficient order update mechanism. In other words, the proposed implementation allows the change of subspace dimension, that is the number of Doppler bins utilised in the post-Doppler detector, at no loss of computational efficiency.

The presented implementation is based on the conjugate directions method which is a well-known iterative method for the solution of system of linear equations [12]. A number of related methods, especially for the solution of reduced-rank Wiener filtering problems, have been proposed in the literature. In [13], the multi-stage Wiener filtering leveraging the computational complexity of the filter with its performance is proposed and these filters have found applications in many different areas burdened with heavy computational requirements, such as code division multiple access (CDMA) multiuser communications and space–time processing [6, 14–16].

Recently, some methods utilising the conjugate gradients for the reduced-rank filtering have been proposed [17, 18]. The methods based on the conjugate gradients are known to produce a solution in the Krylov subspace generated by the clutter and noise covariance matrix ($\boldsymbol{R}_{\text{cn}}$ matrix) and the steering vector. Different from these techniques, we utilise the 'conjugate directions' to solve for the optimal coefficients in the reduced dimensional space. In contrast to the conjugate gradient-based methods, such as [17], the 'conjugate directions' based methods present a solution in a desired subspace which is not necessarily a Krylov space. Recently, the constant false alarm property of the conjugate gradient filters along with some other properties, such as their optimality in the residing Krylov space, have been shown in [17]. The equivalence of the conjugate gradient filters (with a proper initialisation) and the multi-stage Wiener filters has also been shown in [19].

It should be noted that the suggested implementation via conjugate directions is, in principle, equivalent to the

Gram–Schmidt orthogonalisation of the data. Therefore the application of the conjugate directions does not bring any computational advantage in the solution of a system of linear equations; but we show that the process can be indeed advantageous for the order update of the filter used in the post-Doppler detectors. The advantage stems from the connection of the Doppler steering vectors of the post-Doppler detectors with the fast Fourier transformation. That is, for the specific target detection problem the subspace of interest is spanned by the vectors that can be written in the form $[1\ e^{j\phi}\ e^{j2\phi}\ \ldots\ e^{j(N-1)\phi}]$. We show that for this specific subspace, the computational complexity of the suggested order recursive implementation is not more than the conventional implementation and the order recursion feature, which is not available in the conventional implementation, is achieved at no additional computational cost. We believe that such a feature can be very useful in practical applications where the number of available auxiliary vectors can significantly change from cell to cell because of the clutter heterogeneities [20–22].

As noted by Rabideau and Steinhardt [23] the auxiliary vectors that do not have the same statistical characteristics with the cell under test should be discarded in order not to compromise the covariance matrix estimate. Since the number of eliminated auxiliary vectors can change from cell to cell, the order of clutter suppression filter (the subspace dimension) can be altered to match the multiplicity of available auxiliary vectors. An application scenario illustrating the described operation is given in the numerical results section of this paper.

The paper is organised as follows. The next section describes the problem of post-Doppler target detection. The following section presents a conjugate directions based solution to the problem and the computational load of the proposed solution and the conventional solution is compared. In the following section, an application scenario is presented to show the utility of the suggested implementation. The paper concludes with a brief summary and comments.

We briefly summarise the notation adopted in this paper. The superscripts $(\cdot)^{T}$ and $(\cdot)^{H}$ denote the transpose and hermitian operators. The vectors are represented with small case letters with bold italic fonts and assumed to be column vectors, unless otherwise is noted. The matrices are shown with capital bold italic letters. The norm $\|\cdot\|_{\boldsymbol{A}}$ refers to weighted L_2 norm of the argument where $\boldsymbol{A}$ is a positive definite weighting matrix.

2 Problem statement

The detection of known signals with unknown complex coefficients observed under Gaussian noise is the classical radar target detection problem [24]. The problem can be written as follows

$$H_0{:}\boldsymbol{r} = \boldsymbol{n}$$

$$H_1{:}\boldsymbol{r} = \alpha \boldsymbol{s}_{\omega} + \boldsymbol{n}$$

Here $\boldsymbol{n}$ is $N \times 1$ column vector representing the noise. The entries of $\boldsymbol{n}$ are jointly complex Gaussian distributed with zero mean and covariance matrix $\boldsymbol{R}_{\text{cn}}$. We would like to note that the clutter signal is absorbed in $\boldsymbol{n}$, therefore $\boldsymbol{R}_{\text{cn}}$ should be interpreted as the covariance matrix of the clutter and noise. The parameter α represents a complex Gaussian distributed scalar (independent from $\boldsymbol{n}$) with zero mean and variance σ_{α}^2. The vector $\boldsymbol{s}_{\omega}$ is a known vector of dimensions $N \times 1$ representing the signal to be detected

$$\boldsymbol{s}_{\omega} = [1\ e^{j\omega}\ e^{j2\omega}\ \ldots\ e^{j(N-1)\omega}]^{T} \tag{1}$$

The complex exponential vector, given above, represents the Doppler steering vector of a moving target with the Doppler phase shift of ω radians per slow-time sample [25].

The optimal detector, under a fixed false alarm probability, is a linear combiner followed by a magnitude comparator, that is $|\boldsymbol{w}^{H}\boldsymbol{r}| \gtrless \tau$ [25]. It is possible to express the linear combiner as the solution of the following equation system

$$\boldsymbol{R}_{\text{cn}}\boldsymbol{w} = \boldsymbol{s}_{\omega} \tag{2}$$

The optimal combiner is known to maximise the post-combination signal-to-noise-ratio (SNR). From (2), it is clear that the calculation of the optimal detector requires the exact knowledge of $\boldsymbol{R}_{\text{cn}}$; but in many applications, $\boldsymbol{R}_{\text{cn}}$ may not be available. In these applications, the $\boldsymbol{R}_{\text{cn}}$ matrix is estimated using a set of auxiliary vectors

$$\widehat{\boldsymbol{R}}_{\text{cn}} = \frac{1}{L}\sum_{k=1}^{L} \boldsymbol{t}_k \boldsymbol{t}_k^{H} \tag{3}$$

Here we assume that a total of L snapshots of the random vector $\boldsymbol{n}$ are available to the processor as the auxiliary data. Each snapshot is denoted with $\boldsymbol{t}_k$ for $k = \{1, \ldots, L\}$. The auxiliary vectors, that is training vectors, are assumed to collected in the target-free region around the cell of interest. Once an estimate for $\boldsymbol{R}_{\text{cn}}$ is available, the linear combination weights can be calculated by replacing $\boldsymbol{R}_{\text{cn}}$ with its estimate. Owing to the erroneous estimate for the covariance matrix, the weights calculated would not be identical to the optimal weights and hence there will be some loss in the output SNR because of the covariance matrix estimation.

In the work of Reed *et al.* [26], known as the RMB paper, the loss of SNR owing to the covariance matrix estimation errors is examined and its probability density function is presented. The conclusion is that the linear combiner may not be as much effective if the number of snapshots L (the number of auxiliary vectors) is smaller than N (the number of pulses to be combined). More specifically, it has been shown that for the Gaussian distributed snapshot vectors, the SNR loss owing to the estimation of $\boldsymbol{R}_{\text{cn}}$ is on the average $-10\log_{10}[1-(N-1)/(L+1)]$ dB [26] (19). So at least $L = 2N-3$ snapshots are required to have a SNR loss of 3 dB on the average. Unfortunately, in some applications, it is indeed difficult to find sufficient number of auxiliary vectors to guarantee the good performance, especially when N is large.

When the number of auxiliary vectors is not sufficient, one may prefer to reduce N (in effect) to a level that can accommodate the available auxiliary vectors. Such detectors are called the subspace detectors [1, 2]. The subspace detectors reduce the dimensionality of the detection problem. To this purpose, a $\boldsymbol{K} \times \boldsymbol{N}$ matrix $(K < N)$ $\boldsymbol{U}^{H}$ is applied on the received vector $\boldsymbol{r}$, as shown below

$$\underbrace{\boldsymbol{U}^{H}\boldsymbol{r}}_{\widetilde{\boldsymbol{r}}} = \alpha\,\underbrace{\boldsymbol{U}^{H}\boldsymbol{s}_{\omega}}_{\widetilde{\boldsymbol{s}}} + \underbrace{\boldsymbol{U}^{H}\boldsymbol{n}}_{\widetilde{\boldsymbol{n}}} \tag{4}$$

The application of matrix $\boldsymbol{U}^{H}$ on the vector $\boldsymbol{x}$ generates the

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expansion coefficients of $\boldsymbol{x}$ in the subspace spanned by the columns of $\boldsymbol{U}$ matrix. We may consider the vectors with tilde on top as the coefficient vector for the K-dimensional projection of the original vector.

Under these conditions, the observation vector is represented as $\widetilde{\boldsymbol{r}} = \alpha\widetilde{\boldsymbol{s}} + \widetilde{\boldsymbol{n}}$ in the subspace and the optimal combiner for the reduced dimensional problem is the solution of the following equation, $\widetilde{\boldsymbol{R}}_{\text{cn}}\boldsymbol{c} = \widetilde{\boldsymbol{s}}$. Here $\boldsymbol{c}$ is a $K \times 1$ vector representing the linear combiner. Since $\widetilde{\boldsymbol{R}}_{\text{cn}} = \boldsymbol{U}^{\text{H}}\boldsymbol{R}_{\text{cn}}\boldsymbol{U}$ and $\widetilde{\boldsymbol{s}} = \boldsymbol{U}^{\text{H}}\boldsymbol{s}_{\omega}$, the equation system can be written as follows

$$\boldsymbol{U}^{\text{H}}\boldsymbol{R}_{\text{cn}}\boldsymbol{U}\boldsymbol{c} = \boldsymbol{U}^{\text{H}}\boldsymbol{s}_{w} \tag{5}$$

The operation with the linear combiner can then be expressed as

$$\boldsymbol{c}^{\text{H}}\widetilde{\boldsymbol{r}} = \boldsymbol{c}^{\text{H}}\boldsymbol{U}^{\text{H}}\boldsymbol{r} = \underbrace{(\boldsymbol{U}\boldsymbol{c})^{\text{H}}}_{\widehat{\boldsymbol{w}}^{\text{H}}}\boldsymbol{r} \tag{6}$$

Here $\widehat{\boldsymbol{w}} = \boldsymbol{U}\boldsymbol{c}$ is a $N \times 1$ vector representing the optimal linear combination restricted to the given subspace. It should be noted that the subspace in the mentioned problem is spanned by the columns of the matrix $\boldsymbol{U}$ and the optimal combiner belongs to this subspace.

If $\boldsymbol{R}_{\text{cn}}$ matrix is not available, then its estimate $\widehat{\boldsymbol{R}}_{\text{cn}}$ is inserted in (5). The estimation of $\widehat{\boldsymbol{R}}_{\text{cn}}$ and its substitution for $\boldsymbol{R}_{\text{cn}}$ in (5) and the solution of the resultant equation system constitute the conventional method. If the number of auxiliary vectors L is greater than $2K$ (K being the subspace dimension) then the SNR loss owing to imperfection in covariance matrix estimation is limited to 3 dB on the average. Hence by reducing the dimensionality of the problem to a level compatible with the multiplicity of the auxiliary vectors, it can be possible to attain a performance close to the optimal subspace detector utilising the exact $\boldsymbol{R}_{\text{cn}}$, not its estimate.

In the post-Doppler adaptive target detectors, the subspace to be utilised is formed by the Doppler steering vectors. The detection takes place after the application of the discrete Fourier transform (DFT), as the name implies. The subspace for post-Doppler detection can be considered as the span of $\{\boldsymbol{s}_{\omega_1}, \boldsymbol{s}_{\omega_2}, \ldots, \boldsymbol{s}_{\omega_K}\}$, where $\boldsymbol{s}_{\omega_k}$ is as defined in (1).

It should be noted as the subspace dimension increases to N, that is as $K \to N$, the solution in the subspace approaches to the true solution of the equation given in (2), that is $\widehat{\boldsymbol{w}}_K \to \boldsymbol{w}$. It is known that if the signal of interest and the clutter signal are both sufficiently narrowband, the solution in a proper subspace has almost no loss of performance in comparison with the full-dimensional system [9]. In Section 5, we experiment with one-, three- and five-dimensional subspaces for the target detection and show that three- and five-dimensional subspace detectors yield an almost identical performance to the full-dimensional detector (16-dimensional detector). Further details and different types of subspace detectors is given in [9].

We conclude this section with a brief summary. By using the subspace spanned by the columns of $\boldsymbol{U}$ matrix, it is possible to reduce the dimensionality of the detection problem and make an efficient usage of the auxiliary data. A proper subspace for the radar target detection problem is the space spanned by the columns of the $N \times N$ DFT matrix (post-Doppler detection). This subspace captures a significant of amount of total signal energy and retains sufficient degrees of freedom to suppress the clutter. In this paper, we present a conjugate directions based method for the solution of optimal combiner restricted to this subspace. The suggested implementation, different from the conventional one, is order recursive and has a simple mechanism enabling the change of the subspace dimension at no additional computational cost.

3 Conjugate directions based solution

First, we introduce the motivation for the conjugate directions. Our goal is to reproduce the equation system given in (2) as the solution of an optimisation problem. To that purpose, we impose $\widehat{\boldsymbol{w}} = \boldsymbol{U}\boldsymbol{c}$ as a constraint to the following optimisation problem

$$\min J(\boldsymbol{c}) = \min||\boldsymbol{w} - \underbrace{\boldsymbol{U}\boldsymbol{c}}_{\widehat{\boldsymbol{w}}}||^2_{\widehat{\boldsymbol{R}}_{\text{cn}}} \quad \text{such that } \widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{w} = \boldsymbol{s}_{\omega} \tag{7}$$

Here $\boldsymbol{U}$ is a $N \times K$ matrix and $\boldsymbol{c}$ is a $K \times 1$ vector. The cost can be interpreted as the reduction of the residual $\boldsymbol{w} - \widehat{\boldsymbol{w}}$ in the sense of $\widehat{\boldsymbol{R}}_{\text{cn}}$-weighted Euclidean norm, that is $||\boldsymbol{w} - \boldsymbol{U}\boldsymbol{c}||^2_{\widehat{\boldsymbol{R}}_{\text{cn}}} = (\boldsymbol{w} - \boldsymbol{U}\boldsymbol{c})^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}(\boldsymbol{w} - \boldsymbol{U}\boldsymbol{c})$ where $\boldsymbol{w}$ is the optimal linear combiner for the full-dimensional detector.

By taking the gradient of $J(\boldsymbol{c})$ given in (7) and making use of the constraint relation $\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{w} = \boldsymbol{s}_{\omega}$, we can show that the optimal $\boldsymbol{c}$ satisfies $\boldsymbol{U}^{\text{H}}\boldsymbol{R}_{\text{cn}}\boldsymbol{U}\boldsymbol{c} = \boldsymbol{U}^{\text{H}}\boldsymbol{s}_{w}$ which is identical to the equation system given in (5).

The conjugate directions method generates a set of $\widehat{\boldsymbol{R}}_{\text{cn}}$-orthogonal vectors. Two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ are said to be $\widehat{\boldsymbol{R}}_{\text{cn}}$-orthogonal if $\boldsymbol{x}^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{y} = 0$. As shown below, once $\widehat{\boldsymbol{R}}_{\text{cn}}$-orthogonal vectors are generated, the solution top the optimisation problem can be easily generated and the order recursion is immediate.

To show the order recursion, we assume that $\boldsymbol{U}_k$ be a $N \times k$ matrix whose columns span the k-dimensional space of $\{\boldsymbol{s}_{\omega_1}\boldsymbol{s}_{\omega_2}\ldots\boldsymbol{s}_{\omega_k}\}$. At this point, we assume that the coefficients of the optimal filter in the k-dimensional subspace is available to us, that is the solution of the following equation system is given

$$\boldsymbol{U}_k^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{U}_k\boldsymbol{c}_k = \boldsymbol{U}_k^{\text{H}}\boldsymbol{s}_{w} \tag{8}$$

Now we desire to have an order update from k dimensions to $(k+1)$ dimensions. The equation system in $(k+1)$ dimension system can be written as $\boldsymbol{U}_{k+1}^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{U}_{k+1}\boldsymbol{c}_{k+1} = \boldsymbol{U}_{k+1}^{\text{H}}\boldsymbol{s}_{w}$, where $\boldsymbol{U}_{k+1} = [\boldsymbol{U}_k\ \boldsymbol{u}_{k+1}]$. Here $\boldsymbol{u}_{k+1}$ is the column vector concatenated to $\boldsymbol{U}_k$ matrix to increase the subspace dimension to $(k+1)$. The coefficients of the updated filter satisfies the following relation

$$\begin{bmatrix} \boldsymbol{U}_k^{\text{H}} \\ \boldsymbol{u}_{k+1}^{\text{H}} \end{bmatrix}\widehat{\boldsymbol{R}}_{\text{cn}}\begin{bmatrix} \boldsymbol{U}_k & \boldsymbol{u}_{k+1} \end{bmatrix}\boldsymbol{c}_{k+1} = \begin{bmatrix} \boldsymbol{U}_k^{\text{H}} \\ \boldsymbol{u}_{k+1}^{\text{H}} \end{bmatrix}\boldsymbol{s}_{\omega}$$
$$\begin{bmatrix} \boldsymbol{U}_k^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{U}_k & \boldsymbol{U}_k^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{u}_{k+1} \\ \boldsymbol{u}_{k+1}^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{U}_k & \boldsymbol{u}_{k+1}^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{u}_{k+1} \end{bmatrix}\boldsymbol{c}_{k+1} = \begin{bmatrix} \boldsymbol{U}_k^{\text{H}}\boldsymbol{s}_{\omega} \\ \boldsymbol{u}_{k+1}^{\text{H}}\boldsymbol{s}_{\omega} \end{bmatrix} \tag{9}$$

The central observation is that $\boldsymbol{u}_k$ vectors have conjugate directions, that is $\boldsymbol{u}_{k+1}^{\text{H}}\widehat{\boldsymbol{R}}_{\text{cn}}\boldsymbol{u}_l = 0\ \forall l = \{1, 2, \ldots, k\}$. The

equation system reduces to the following one

$$\begin{bmatrix} \boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k & \boldsymbol{0}_{K\times 1} \\ \boldsymbol{0}_{1\times K} & \boldsymbol{u}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1} \end{bmatrix} \boldsymbol{c}_{k+1} = \begin{bmatrix} \boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{s}_{\omega} \\ \boldsymbol{u}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega} \end{bmatrix} \tag{10}$$

From the last equation system, the solution can be immediately written as follows

$$\boldsymbol{c}_{k+1} = \begin{bmatrix} \boldsymbol{c}_k \\ \dfrac{\boldsymbol{u}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}}{\boldsymbol{u}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1}} \end{bmatrix} \tag{11}$$

To summarise if the subspace of interest can be $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthogonalised (or preferably $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormalised), the optimal linear combiner is found almost effortlessly and the order recursion is evident from the scheme.

If we denote the $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormal basis vectors for the subspace of $\{\boldsymbol{s}_{\omega_1}, \boldsymbol{s}_{\omega_2}, \ldots, \boldsymbol{s}_{\omega_k}\}$ with $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ and replace $\boldsymbol{U}_k$ in (8) with $\boldsymbol{V}_k$ ($\boldsymbol{V}_k = [\boldsymbol{v}_1\ \boldsymbol{v}_2\ \ldots\ \boldsymbol{v}_k]$); the equation system given in (8) reduces to

$$\underbrace{\boldsymbol{V}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{V}_k}_{\boldsymbol{I}}\boldsymbol{c}_k = \boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{s}_{\omega} \tag{12}$$

and the optimal linear combiner in this subspace becomes $\widehat{\boldsymbol{w}} = \boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{s}_{\omega}$. This relation can also be written as

$$\widehat{\boldsymbol{w}}_{k+1} = \widehat{\boldsymbol{w}}_k + \boldsymbol{v}_{k+1}\boldsymbol{v}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega} \tag{13}$$

explicitly showing the order recursion.

Another order-recursive property of the conjugate directions based solution is that one can reduce the dimensionality of the filter at no cost. Once the R-orthonormal basis vectors are generated, one can easily utilise some subset of available vectors. However, for the conventional method [sample matrix inversion (SMI)], the dimensionality reduction is achieved after rebuilding the matrix for the linear system and its inversion.

The process of finding an $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormal basis, that is $\boldsymbol{v}_k$ vectors, is closely related to the Gram–Schmidt orthogonalisation [12]. Unfortunately, the computational load of this method is equivalent to the matrix inversion, thus there is no computational gain with the $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormalisation in general. Readers can compare the steps of procedure presented in Table 1 with the Gram–Schmidt orthogonalisation.

The subspace of interest can be explicitly written as span$\{\boldsymbol{s}_{\omega_1}, \boldsymbol{s}_{\omega_2}, \ldots, \boldsymbol{s}_{\omega_K}\}$. Furthermore, the space is assumed to be spanned by the columns of DFT matrix. In other words, it is assumed that $\omega_k - \omega_l$ is an integer multiple of $(2\pi/N)$. Then the subspace of interest is spanned by a subset of K columns of $N \times N$ DFT matrix. For this specific subspace and for $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$ given in (3), the conjugate directions has no additional load as shown in the next section.

Table 1 Conjugate directions method solution of $\boldsymbol{Ax} = \boldsymbol{b}$ system

$\{u_1, u_2, \ldots, u_K\}$:	a set of linearly independent vectors
A:	Hermitian $N \times N$ matrix
b:	$N \times 1$ vector
1:	initialise, $k = 1$, $\widehat{\boldsymbol{v}}_1 = \boldsymbol{u}_1$, $\boldsymbol{v}_1 = \widehat{\boldsymbol{v}}_1/\widehat{\boldsymbol{v}}_1^{\mathrm{H}}\boldsymbol{A}\widehat{\boldsymbol{v}}_1$
2:	do
3:	$k \Leftarrow k+1$
4:	$\widehat{\boldsymbol{v}}_k = \boldsymbol{u}_k - \Sigma_{i=1}^{k-1}\boldsymbol{v}_i\dfrac{\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{u}_k}{\boldsymbol{v}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{v}_i}$: $\boldsymbol{A}$-orthogonal vec.
5:	$\boldsymbol{v}_k = \widehat{\boldsymbol{v}}_k/\widehat{\boldsymbol{v}}_k^{\mathrm{H}}\boldsymbol{A}\widehat{\boldsymbol{v}}_k$: $\boldsymbol{A}$-orthonormal vec.
6:	while $k \le K$
7:	$\boldsymbol{V} = [\boldsymbol{v}_1\ \boldsymbol{v}_2 \ldots \boldsymbol{v}_K]$
8:	$\widehat{\boldsymbol{x}} = \boldsymbol{V}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{b}$

4 Computational load comparison of conventional and conjugate directions based subspace filtering

The total computational cost of evaluating the optimal linear combiner $\boldsymbol{w}$ in the k-dimensional subspace of DFT vectors using both the conventional SMI and conjugate directions approach is given. Moreover, the cost of updating the solution by increasing the subspace dimension by one is also discussed.

The cost calculation is divided into two steps. In the construction step, the cost of the constructing the matrix from the training vectors is given. In the order update step, the cost of order update is given. Some comparisons are provided at the conclusion of this section.

4.1 Construction cost

4.1.1 Conventional method: The linear combiner in the k-dimensional subspace $\boldsymbol{U}_k$ can be expressed as $\hat{\boldsymbol{w}}_k = \boldsymbol{U}_k\boldsymbol{c}_k$. The optimal $\boldsymbol{c}_k$ that minimises the aforementioned cost function is the solution of the linear system $\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k\boldsymbol{c} = \boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{s}_w$. One should note that $\boldsymbol{U}_k = [\boldsymbol{s}_{w_1} \ldots \boldsymbol{s}_{w_k}]$ is a $N \times k$ matrix, columns of which are DFT vectors. To solve this system one should first evaluate $\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k$ and $\boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{s}_w$ and then should solve the linear system for $\boldsymbol{c}$. Furthermore, let us assume that $\mathrm{DFT}\{\boldsymbol{t}_i\} = \boldsymbol{U}_N^{\mathrm{H}}\boldsymbol{t}_i = \boldsymbol{T}_i$ has been precomputed for $\{\boldsymbol{t}_i\}_{i=1}^{L}$ at the cost of $L(N/2)\log_2 N$ and $\boldsymbol{U}_N^{\mathrm{H}}\boldsymbol{s}_{\omega} = \mathrm{DFT}\{\boldsymbol{s}_{\omega}\}$ has been precomputed at the cost of $(N/2)\log_2 N$. Evaluation of $\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k$ is as follows

$$\begin{aligned} \boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k &= \boldsymbol{U}_k^{\mathrm{H}}\frac{1}{L}\sum_{i=1}^{L}\boldsymbol{t}_i\boldsymbol{t}_i^{\mathrm{H}}\boldsymbol{U}_k \\ &= \frac{1}{L}\sum_{i=1}^{L}(\boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{t}_i)(\boldsymbol{t}_i^{\mathrm{H}}\boldsymbol{U}_k) \\ &= \frac{1}{L}\sum_{i=1}^{L}\underbrace{\boldsymbol{T}_i[w_1, \ldots, w_k]}_{k\times 1}\boldsymbol{T}_i[w_1, \ldots, w_k]^{\mathrm{H}} \end{aligned} \tag{14}$$

Each vector–vector multiplication inside the summation requires k^2 operations, hence the evaluation of $\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k$ requires k^2L multiplications. The matrix–vector product $\boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{s}_w$ has been computed at the FFT stage so it comes at no cost. The solution of the linear system for $\boldsymbol{c}_k$ requires k^3 multiplications. The linear combiner $\hat{\boldsymbol{w}}_k = \boldsymbol{U}_k\boldsymbol{c}_k$ can be computed with additional Nk multiplications. Therefore the total cost of this method is $(L+1)(N/2)\log_2 N + k^3 + k^2L + kN$.

4.1.2 Conjugate directions method: The main computational complexity of the proposed method is due to the $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormalisation of the basis vectors $\{\boldsymbol{s}_{\omega_1}, \ldots, \boldsymbol{s}_{\omega_k}\}$. The computational cost of this orthonormalisation can be proved inductively. One important thing to notice in order to understand the proposed solution is that the $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormal

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set $\{\boldsymbol{v}_i\}_{i=1}^{k}$ spans the same subspace that is spanned by $\{\boldsymbol{s}_{\omega_1}, \ldots, \boldsymbol{s}_{\omega_k}\}$. Therefore one can express the $\boldsymbol{v}_i$'s as

$$\boldsymbol{v}_i = \sum_{n=1}^{k} \alpha_i^n \boldsymbol{s}_{\omega_n} \tag{15}$$

Let us assume that the DFTs of $\{\boldsymbol{t}_i\}_{i=1}^{L}$ and $\boldsymbol{s}_\omega$ have been precomputed at the cost of $(L+1)(N/2)\log_2 N$. The Gram–Schmidt process is as follows:

- *Step 1:*

$$\begin{aligned}\boldsymbol{v}_1 &= \frac{\boldsymbol{s}_{\omega_1}}{\|\boldsymbol{s}_{\omega_1}\|_{\hat{\boldsymbol{R}}_{\text{cn}}}}, \quad \text{where } \left\|\boldsymbol{s}_{\omega_1}\right\|_{\hat{\boldsymbol{R}}_{\text{cn}}}^2 = \boldsymbol{s}_{\omega_1}^{\text{H}} \widehat{\boldsymbol{R}}_{\text{cn}} \boldsymbol{s}_{\omega_1} \\ &= \underbrace{\frac{1}{L}\sum_{i=1}^{L} |\boldsymbol{T}_i[\omega_1]|^2}_{L \text{ multiplications}}\end{aligned} \tag{16}$$

Therefore $\boldsymbol{v}_1 = \alpha_1^1 \boldsymbol{s}_{\omega_1}$, where α_1^1 is as above and is computed at cost of L multiplications.

- *Step 2:*

$$\begin{aligned}\boldsymbol{v}_2 &= \boldsymbol{s}_{\omega_2} - (\boldsymbol{s}_{\omega_2}^{\text{H}} \widehat{\boldsymbol{R}}_{cn} \boldsymbol{v}_1)\boldsymbol{v}_1 \\ &= \boldsymbol{s}_{\omega_2} - (\alpha_1^1)^2 (\boldsymbol{s}_{\omega_2}^{\text{H}} \widehat{\boldsymbol{R}}_{\text{cn}} \boldsymbol{s}_{\omega_1}) \boldsymbol{s}_{\omega_1} \\ &= \boldsymbol{s}_{\omega_2} + \beta_1^1 \boldsymbol{s}_{\omega_1}, \quad \text{where } \beta_1^1 = \underbrace{-\frac{1}{L}(\alpha_1^1)^2 \sum_{i=1}^{L} \boldsymbol{T}_i[\omega_2]\boldsymbol{T}_i[\omega_1]^*}_{L \text{ multiplications}}\end{aligned} \tag{17}$$

- *Step 3:*

$$\begin{aligned}\boldsymbol{v}_2 &= \frac{\boldsymbol{v}_2}{\|\boldsymbol{v}_2\|_{\hat{\boldsymbol{R}}_{\text{cn}}}}, \quad \text{where} \\ \left\|\boldsymbol{v}_2\right\|_{\hat{\boldsymbol{R}}_{\text{cn}}}^2 &= (\boldsymbol{s}_{\omega_2} + \beta_1^1 \boldsymbol{s}_{\omega_1})^{\text{H}} \widehat{\boldsymbol{R}}_{cn} (\boldsymbol{s}_{\omega_2} + \beta_1^1 \boldsymbol{s}_{\omega_1})\end{aligned} \tag{18}$$

(see (19))

Therefore $\boldsymbol{v}_2 = \alpha_2^2 \boldsymbol{s}_{\omega_2} + \alpha_2^1 \boldsymbol{s}_{\omega_1}$.

- *Step 4:* Repeat the Gram–Schmidt orthonormalisation using the same bookkeeping methodology.

One can show by induction that evaluation of the kth $\widehat{\boldsymbol{R}}_{\text{cn}}$-orthonormal vector requires additional kL multiplications assuming that $\{\boldsymbol{v}_k\}_{i=1}^{k-1}$ exists. The main rationale is that kth step of Gram–Schmidt requires the inner products $\{< \boldsymbol{s}_{\omega_i}, \boldsymbol{s}_{\omega_j} >_{\hat{\boldsymbol{R}}_{\text{cn}}}\}_{i,\, j \in \{1,\ldots,k\}}$ and the inner products $\{< \boldsymbol{s}_{\omega_i}, \boldsymbol{s}_{\omega_j} >_{\widehat{\boldsymbol{R}}_{\text{cn}}}\}_{i,j \in \{1,\ldots,k-1\}}$ exists from the earlier steps of the process. Hence, the only inner products to be computed at the kth step is the inner products $\{< \boldsymbol{s}_{\omega_k}, \boldsymbol{s}_{\omega_j} >_{\widehat{\boldsymbol{R}}_{\text{cn}}}\}_{j \in \{1,\ldots,k\}}$, which requires kL multiplications.

The Gram–Schmidt procedure returns the coefficients $\{\alpha_k^n\}$ for the $\widehat{\boldsymbol{R}}_{\text{cn}}$-orthonormalised set $\{\boldsymbol{v}_i\}_{i=1}^{k}$ at the cost of $[k(k+1)/2]L$. Given these coefficients, one can evaluate the k-dimensional approximation of the linear combiner as (see equation at the bottom of the page)

The last part of the multiplication above has been computed at the FFT stage, hence it comes at no cost. Moreover, since the coefficient matrices are lower and upper triangular, each matrix–vector multiplication costs $[k(k+1)/2]$. Multiplication of $\begin{bmatrix}\boldsymbol{s}_{\omega_1} & \boldsymbol{s}_{\omega_2} & \ldots & \boldsymbol{s}_{\omega_k}\end{bmatrix}$ with the $k \times 1$ resultant vector leads to Nk multiplications. Therefore evaluation of the linear combiner costs $Nk + 2(k(k+1)/2) = k(k+N+1)$.

Therefore the total cost of matrix construction is

$$\begin{aligned}&(L+1)(N/2)\log_2 N + (k(k+1)/2)L + k(k+N+1) \\ &= (L+1)(N/2)\log_2 N + k^2((L/2)+1) + k((L/2)+N+1)\end{aligned}$$

4.2 Update cost

4.2.1 Conventional method: The linear combiner $\boldsymbol{w}$ in the $k+1$-dimensional subspace spanned by the columns of $\boldsymbol{U}_{k+1}$, where $\boldsymbol{U}_{k+1} = [\boldsymbol{U}_k \;\; \boldsymbol{u}_{k+1}]$ can be expressed as $\boldsymbol{w} \simeq \boldsymbol{U}_{k+1}\boldsymbol{c}_{k+1}$ and the optimal $\boldsymbol{c}$ that minimises the cost

$$\begin{aligned}\left\|\boldsymbol{v}_2\right\|_{\hat{\boldsymbol{R}}_{\text{cn}}}^2 &= \underbrace{\boldsymbol{s}_{\omega_2}^{\text{H}} \widehat{\boldsymbol{R}}_{\text{cn}} \boldsymbol{s}_{\omega_2}}_{} + |\beta_1^1|^2 \underbrace{\boldsymbol{s}_{\omega_1}^{\text{H}} \widehat{\boldsymbol{R}}_{\text{cn}} \boldsymbol{s}_{\omega_1}}_{\text{computed at step 1}} + 2\text{Re}\underbrace{\{\beta_1^1 \boldsymbol{s}_{\omega_2}^{\text{H}} \widehat{\boldsymbol{R}}_{\text{cn}} \boldsymbol{s}_{\omega_1}\}}_{\text{computed at step 2}} \\ &\quad \times \underbrace{\frac{1}{L}\sum_{i=1}^{L} |\boldsymbol{T}_i[\omega_2]|^2}_{L \text{ multiplications}}\end{aligned} \tag{19}$$

$$\begin{aligned}\hat{\boldsymbol{w}}_k &= \boldsymbol{V}_k \boldsymbol{V}_k^{\text{H}} \boldsymbol{s}_\omega = \begin{bmatrix}\boldsymbol{v}_1 & \boldsymbol{v}_2 & \ldots & \boldsymbol{v}_k\end{bmatrix} \begin{bmatrix}\boldsymbol{v}_1^{\text{H}} \\ \boldsymbol{v}_2^{\text{H}} \\ \vdots \\ \boldsymbol{v}_k^{\text{H}}\end{bmatrix} \boldsymbol{s}_\omega \\ &= \underbrace{\begin{bmatrix}\boldsymbol{s}_{\omega_1} & \boldsymbol{s}_{\omega_2} & \ldots & \boldsymbol{s}_{\omega_k}\end{bmatrix}}_{N \times k} \begin{bmatrix}\alpha_1^1 & \alpha_2^1 & \ldots & \alpha_k^1 \\ 0 & \alpha_2^2 & \ldots & \alpha_k^2 \\ 0 & 0 & \ldots & \alpha_k^3 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \alpha_k^k\end{bmatrix} \begin{bmatrix}(\alpha_1^1)^* & 0 & 0 & \ldots & 0 \\ (\alpha_2^1)^* & (\alpha_2^2)^* & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ (\alpha_k^1)^* & (\alpha_k^2)^* & (\alpha_k^3)^* & \ldots & (\alpha_k^k)^*\end{bmatrix} \underbrace{\begin{bmatrix}\boldsymbol{s}_{\omega_1}^{\text{H}} \\ \boldsymbol{s}_{\omega_2}^{\text{H}} \\ \ldots \\ \boldsymbol{s}_{\omega_k}^{\text{H}}\end{bmatrix} \boldsymbol{s}_\omega}_{\boldsymbol{s}_\omega[\omega_1,\ldots,\omega_k]}\end{aligned}$$

function satisfies

$$\boldsymbol{U}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_{k+1}\boldsymbol{c}_{k+1}=\boldsymbol{U}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega} \tag{20}$$

$$\begin{bmatrix}\boldsymbol{U}_k^{\mathrm{H}}\\ \boldsymbol{u}_{k+1}^{\mathrm{H}}\end{bmatrix}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\begin{bmatrix}\boldsymbol{U}_k & \boldsymbol{u}_{k+1}\end{bmatrix}\boldsymbol{c}_{k+1}=\begin{bmatrix}\boldsymbol{U}_k^{\mathrm{H}}\\ \boldsymbol{u}_{k+1}^{\mathrm{H}}\end{bmatrix}\boldsymbol{s}_{\omega}$$

$$\underbrace{\begin{bmatrix}\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k & \boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1}\\ \boldsymbol{u}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{U}_k & \boldsymbol{u}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1}\end{bmatrix}}_{\star}\boldsymbol{c}_{k+1}=\begin{bmatrix}\boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{s}_{\omega}\\ \boldsymbol{u}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}\end{bmatrix} \tag{21}$$

The evaluation of $\boldsymbol{U}_k^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1}$, $\boldsymbol{u}_{k+1}^{\mathrm{H}}\widehat{\boldsymbol{R}}_{\mathrm{cn}}\boldsymbol{u}_{k+1}$ and $\boldsymbol{u}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}$ requires kL, L and zero (precomputed at FFT stage) multiplications, respectively. Hence, forming the $k+1$-dimensional linear system on top of the previous k-dimensional linear system requires additional $(k+1)$ L multiplications. Direct inversion of the matrix $\star$ requires $(k+1)^3$ multiplications. (It is important to note that, one can apply the matrix inversion lemma to the matrix $\star$ in order to have a recursive formula for $\boldsymbol{c}_{k+1}$ in terms of $\boldsymbol{c}_k$. The cost of this update via matrix inversion lemma will still require $3k^2$ more multiplications and to achieve this computational level, some bookkeeping is necessary.) To find the new linear combiner $\hat{\boldsymbol{w}}_{k+1}=\boldsymbol{U}_{k+1}\boldsymbol{c}_{k+1}$ one needs extra $N(k+1)$ multiplications. Therefore update via the conventional method requires $N(k+1)+(k+1)L+3k^2$ multiplications.

4.2.2 Conjugate directions method: The linear combiner in the $k+1$-dimensional subspace spanned by $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormal vectors $\{\boldsymbol{v}_i\}_{i=1}^{k+1}$ can be expressed as $\hat{\boldsymbol{w}}_{k+1}=\boldsymbol{V}_{k+1}\boldsymbol{V}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}$, where $\boldsymbol{V}_{k+1}=[\boldsymbol{V}_k\ \boldsymbol{v}_{k+1}]$. Note that k-dimensional approximation is expressed as $\hat{\boldsymbol{w}}_k=\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{s}_{\omega}$. Hence, one can express the update as (see (22))

The rightmost matrix–vector multiplication has been precomputed at the FFT stage, so it comes at no cost. The cost of this update is $N(k+1)+2(k+1)=(N+2)(k+1)$ multiplications for evaluating $\boldsymbol{v}_{k+1}\boldsymbol{v}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}$ and $(k+1)$ L for evaluating the $(k+1)$th $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthonormal vector, using the already available set $\{\boldsymbol{v}_i\}_{i=1}^{k}$. Thus, total update cost for the conjugate directions based solution is $(k+1)(N+L+2)$.

4.3 Comparison

Table 2 summarises the computational complexity for conventional method and the suggested method. When the computational cost of the conventional approach is compared with the proposed one, the proposed one has the savings of $O(K^3)$ operations which can be significant for large subspace dimensions. It should be remembered that for the target detection application the number of pulses emitted can be a large number ($N>50$) and K, the dimension of solution subspace, is assumed to be much smaller than N. Typically, the snapshot number L is around 3–5 K for an acceptable performance [26]. Owing to this, we do not consider the savings of $O(K^3)$ multiplications as the main advantage of proposed implementation, instead we would like to underline the possibility of order recursion. In the following section, an application example is given to examine the interplay of the parameters N, K and L in a practical scenario.

5 Numerical results

In this section, we describe a scenario for the application of the order recursive, subspace-based adaptive target detector. We consider a pulse Doppler radar with a rotary antenna. The radar operates at a pulse repetition frequency (PRF) of 2500 Hz and transmitting 16 pulses in a dwell. The antenna rotates at the rate of 40 revolutions per minute. For simplicity, it is assumed that the radar is a land-based system and the clutter decorrelation is assumed to be dominated by the antenna rotation, that is the antenna scanning modulation. The azimuth beam pattern of the antenna is assumed to be in the form of Gaussian shape with the 3 dB beamwidth of 2°. Other system parameters are given in Table 3.

Table 3 System parameters

Antenna parameters	
beamwidth, BW	2°
antenna revolution rate	40 rpm
Signal and clutter parameters	
PRF	2500 Hz
number of pulses (N_{pulse})	16
number of aux. vectors, L	5 and 10
SNR	10 dB
CNR	50 dB
Jammer parameters	
JNR	15 dB
jammed Doppler freq.	1250 Hz (after folding)

Table 2 Computation cost comparison of conventional method and suggested method

	Number of multiplications	
	Conventional method	Suggested method
construction step	$(L+1)(N/2)\log_2 N+k^3+k^2L+kN$	$(L+1)(N/2)\log_2 N+k^2((L/2)+1)+k((L/2)+N+1)$
update step	$N(k+1)+(k+1)L+3k^2$	$(k+1)(N+L+2)$

$$\begin{aligned}\hat{\boldsymbol{w}}_{k+1}&=\hat{\boldsymbol{w}}_k+\boldsymbol{v}_{k+1}\boldsymbol{v}_{k+1}^{\mathrm{H}}\boldsymbol{s}_{\omega}\\ &=\hat{\boldsymbol{w}}_k+\underbrace{\begin{bmatrix}\boldsymbol{s}_{\omega_1} & \cdots & \boldsymbol{s}_{\omega_{k+1}}\end{bmatrix}}_{N\times(k+1)}\begin{bmatrix}\alpha_{k+1}^1\\ \vdots\\ \alpha_{k+1}^{k+1}\end{bmatrix}\begin{bmatrix}(\alpha_{k+1}^1)^* & \cdots & (\alpha_{k+1}^{k+1})^*\end{bmatrix}\underbrace{\begin{bmatrix}\boldsymbol{s}_{\omega_1}^{\mathrm{H}}\\ \vdots\\ \boldsymbol{s}_{\omega_{k+1}}^{\mathrm{H}}\end{bmatrix}\boldsymbol{s}_{\omega}}_{\boldsymbol{s}_{\omega}[\omega_1,\ldots,\omega_{k+1}]}\end{aligned} \tag{22}$$

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From the given numerical values, the clutter auto-correlation sequence can be written as $r_c(k) = \sigma_c^2 \rho^{k^2}$, where $\rho = 0.9936$. Here σ_c^2 denotes the clutter power. In addition to the clutter, one of the DFT bins is also assumed to be jammed with the jamming-to-noise ratio (JNR) of 30 dB as shown in Table 3.

For the comparison of different detectors, the improvement factor (IF) is presented as the figure of merit. The improvement factor is defined as the ratio of the signal-to-clutter-noise ratio (SCNR) before and after processing. To simplify the description and notation, we consider the combination of jamming and clutter signals as the interference and denote the combination as clutter with some abuse of terminology. In other words, the word clutter refers to the combination of the actual clutter signal (centred around DC frequency) and the jamming signal (centred around 1250 Hz).

Fig. 1 presents the improvement factor for different detectors. The detectors presented are the optimal filter and the subspace filters of dimensions 1, 3, 5 and 11. The single-dimensional subspace detector is a 16×1 vector which is in the subspace of span $\{\boldsymbol{s}_\omega\}$. Here ω corresponds to the hypothesised Doppler frequency of the target as in (1). It should be noted that the one-dimensional subspace filter is equivalent to the matched filter for the hypothesised Doppler frequency.

The three-dimensional subspace is formed by the span of $\{\boldsymbol{s}_{\omega-(2\pi/N)}, \boldsymbol{s}_\omega, \boldsymbol{s}_{\omega+(2\pi/N)}\}$. It should be noted that this subspace is formed by the target steering vector and two closest DFT bins neighbouring the steering vector. Similarly, the five-dimensional subspace includes the steering vector and four nearest DFT bins, that is $\{\boldsymbol{s}_{\omega-(4\pi/N)}, \boldsymbol{s}_{\omega-(2\pi/N)}, \boldsymbol{s}_\omega, \boldsymbol{s}_{\omega+(2\pi/N)}, \boldsymbol{s}_{\omega+(4\pi/N)}\}$. The 11-dimensional filter is defined similarly. The 16-dimensional subspace filter spans the whole space and is the SCNR maximising optimal filter.

Fig. 1 shows the variation of the improvement factor when the clutter covariance matrix is perfectly known. The left panel of Fig. 1 shows the complete range for the target Doppler frequency and the right panel shows the spectrum around the jammed frequency. In this figure, the subspace filters are constructed error-free and the performance gap between the optimal filter and the subspace filters is solely owing to the subspace limitations.

In Fig. 1, an upper bound for the improvement factor is also provided in addition to the performance curves. This bound shows the achievable improvement factor for the case of total clutter suppression and coherent integration of the target signal. It should be noted that the upper bound is almost achieved with the optimal filter for the Doppler frequencies at a sufficient distance from the clutter and jamming centres, that is DC frequency and the frequency of 1250 Hz.

Fig. 1 shows that the one-dimensional filter is incapable of providing any clutter suppression, whereas other sub-space filters successfully handle the clutter suppression. Fig. 1*b* shows that the performance of three- and five-dimensional subspaces differs by a maximum of 2.5 dB around the jammed frequency. The 11-dimensional filter and the higher dimensional ones are virtually indistinguishable from the optimal filter.

Fig. 2 presents the improvement factor curves when the clutter covariance matrix is estimated using $L = 5$ auxiliary vectors. The performance of 16-dimensional subspace filter with the estimated covariance matrix is very poor, even poorer than the matched filter, that is one-dimensional subspace filter. The performance drop of the 16-dimensional filter is due to the limited amount of auxiliary data for the estimation of clutter covariance matrix. It should be remembered that 16-dimensional filter is the optimal for the perfectly known covariance matrix and the performance of this filter severely degrades in the presence of estimation errors. It can be noted from Fig. 2 that the three-dimensional subspace filter presents a better improvement factor than other filters when the auxiliary data are limited to $L = 5$ vectors. The performance drop of the other filters is solely because of the insufficient amount of auxiliary vectors for clutter covariance estimation.

The next figure shows a similar comparison when the clutter covariance matrix is estimated using $L = 15$ auxiliary vectors. As can be noted from Fig. 3, the performances of the optimal filter for the known covariance

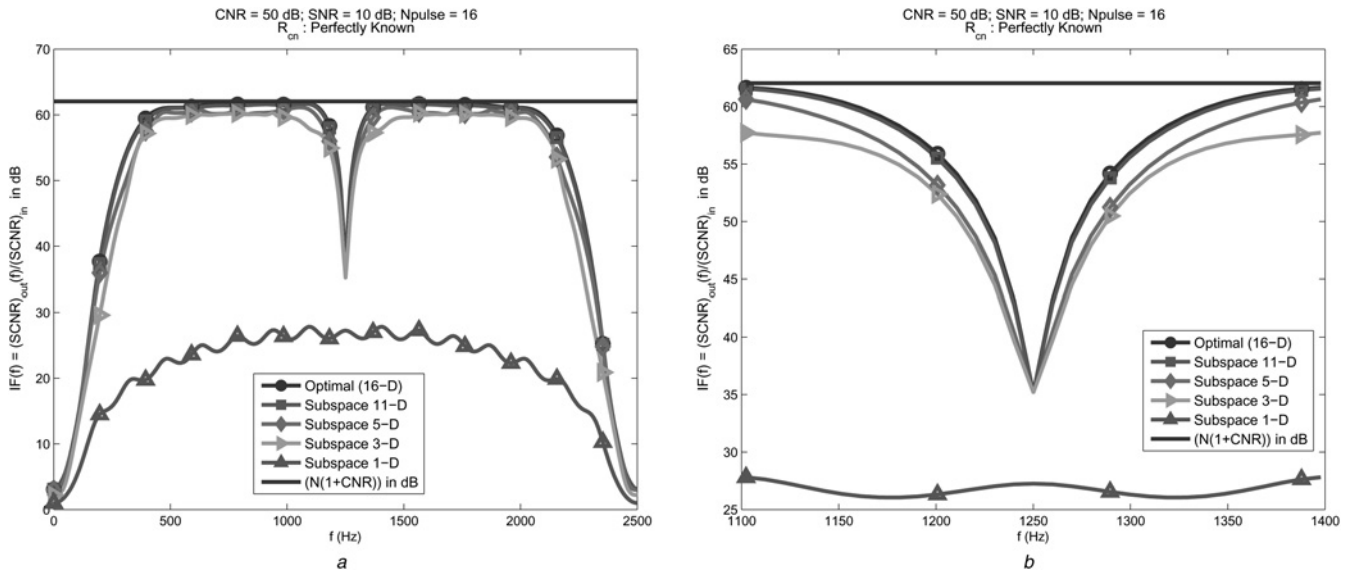

Fig. 1 *Improvement factor for known covariance matrix*

a Complete spectrum
b Spectrum around jammed frequency

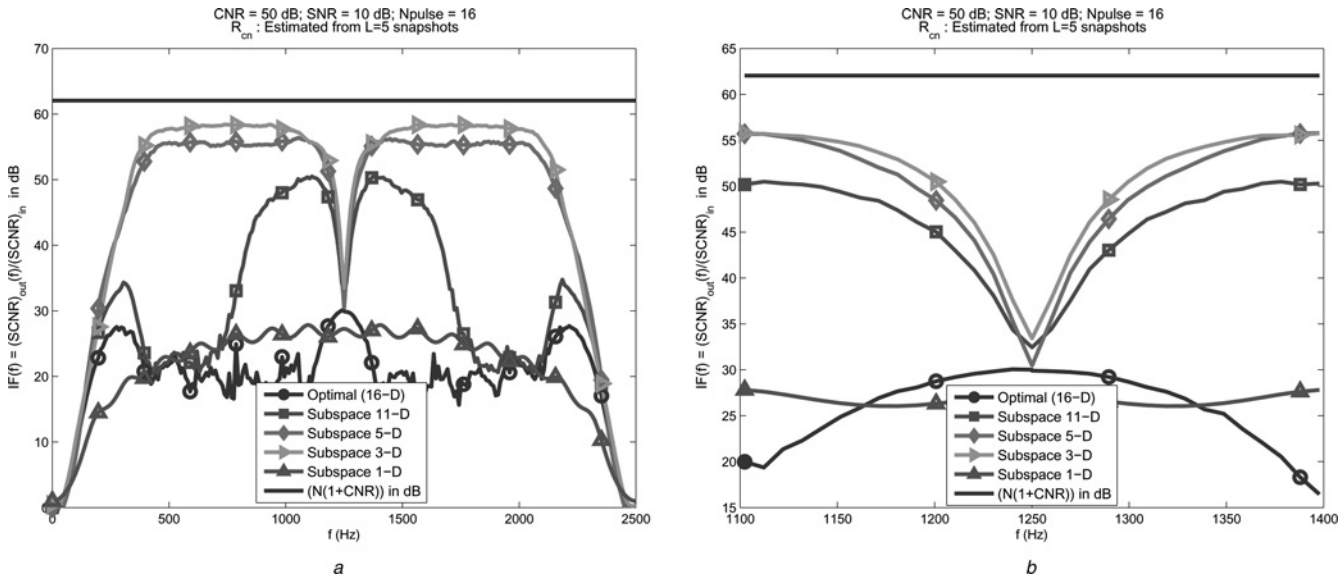

Fig. 2 *Improvement factor for the estimated covariance matrix with $L = 5$ snapshots*

a Complete spectrum
b Spectrum around jammed frequency

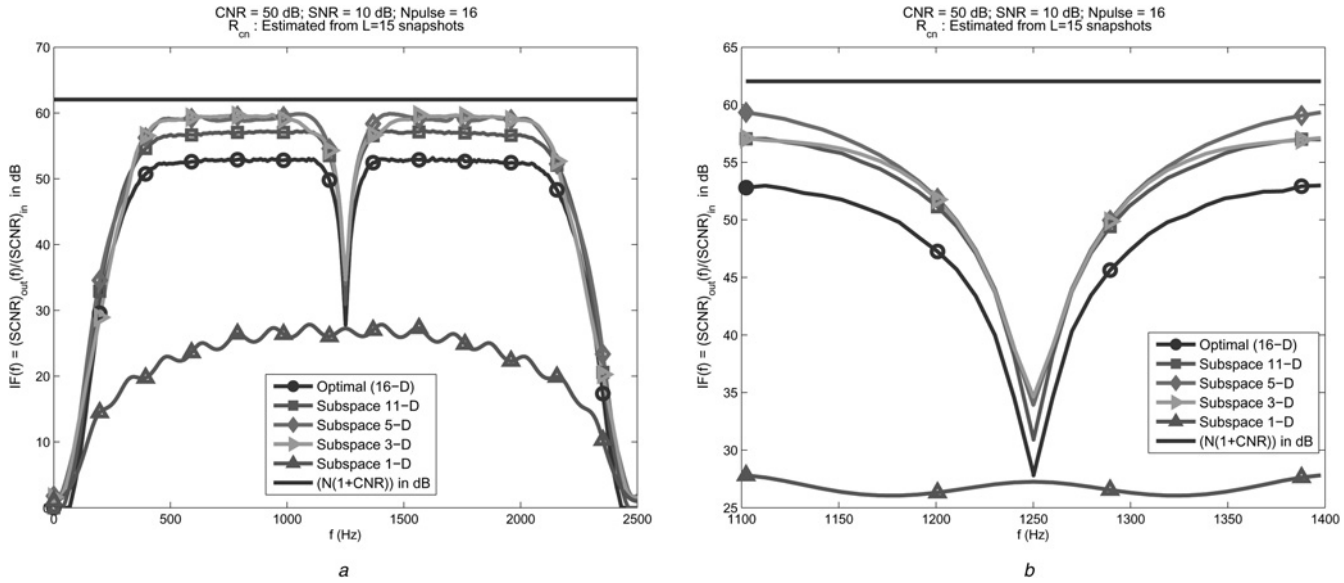

Fig. 3 *Improvement factor for the estimated covariance matrix with $L = 15$ snapshots*

a Complete spectrum
b Spectrum around jammed frequency

case (16-dimensional subspace filter) and the 11-dimensional filter are much better in comparison to the case of covariance matrix estimation with $L = 5$ vectors. (It should be noted that the performance curves for higher dimensional filters lie in between the 11-dimensional and 16-dimensional filter.) In spite of the mentioned improvement, both filters cannot meet the performance of the three-dimensional and the five-dimensional filters for $L = 15$. Hence, the usage of the high-dimensional filters should not be preferable unless the number of auxiliary vectors is exceptionally high.

It should be noted that five-dimensional subspace filter has a better improvement factor than three-dimensional subspace filter especially around the jammed frequency. Hence the utilisation of more auxiliary vectors enhances the performance of the five-dimensional filter most. A further increase in the number of auxiliary vectors can potentially capture 2.5 dB of gain as can be noted from Fig. 1.

A similar comparison is given to examine the effect of clutter spread owing to the antenna rotation. The figure on the left side of Fig. 4 shows the improvement factor curves in the presence of secondary jammer at the frequency of PRF/4, that is 625 Hz. All other parameters, including the antenna rotation of 40 rpm, are as in Table 3. The figure on the right side of Fig. 4 shows the performance curves when the antenna rotation rate is increased to 60 rpm. With $L = 15$ training vectors, the five-dimensional filter performs better than the other filters as in Fig. 3. It can also be noted that the increased clutter spread for the rate 60 rpm causes up to 6 dB of performance loss.

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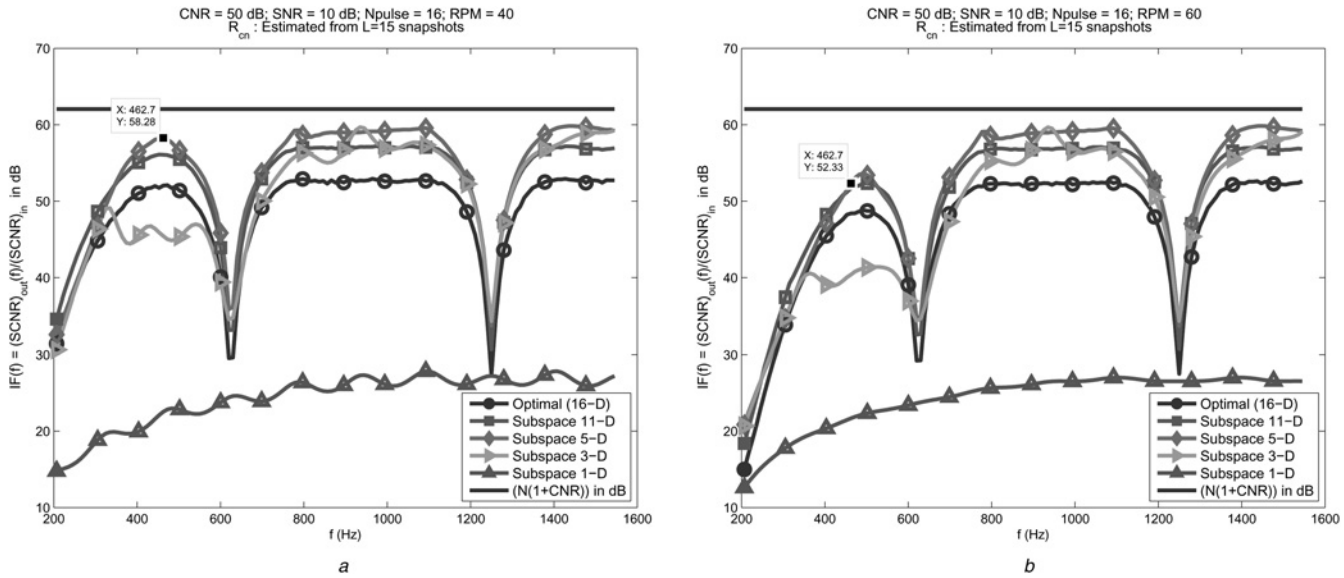

Fig. 4 *Improvement factor for the estimated covariance matrix with L = 15 snapshots at two different antenna rotation rates*

a Spectrum around jammed frequencies (rpm = 40)
b Spectrum around jammed frequencies (rpm = 60)

The presented example shows that depending on the amount of auxiliary data, the subspace dimension should be changed to increase the effectiveness of clutter suppression. As noted by Rapideau and Steinhardt [23], for an effective clutter suppression a suitable subset of auxiliary vectors should be utilised in the covariance matrix estimation. The auxiliary vectors containing powerful clutter discretes should be discarded, whereas the auxiliary vectors sharing the common statistical characteristics with the cell of interest should be utilised for the covariance matrix estimation. The elimination of auxiliary vectors can lead to a varying number of auxiliary vectors for every cell and this may necessitate an adjustment for the subspace dimension of the post-Doppler filter as noted in this section. Such a change can also be necessitated by the variation of CNR and JNR values in space and time. Therefore it is highly unlikely that a single subspace filter would be effective for the clutter suppression at all operational scenarios. The suggested conjugate directions based implementation allows an almost effortless change of the subspace dimension without any loss of computational efficiency and hence for the applications requiring frequent subspace updates, the required calculations can be done at a low cost using the order recursive scheme proposed.

6 Conclusions

In this paper, we present a novel implementation for the post-Doppler adaptive target detectors. Post-Doppler target detectors are frequently utilised to detect slow moving targets embedded in non-stationary clutter which is an especially difficult task for the airborne radar systems.

The implementation is based on an alternative interpretation of the subspace filters through $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$-orthogonal vectors where $\widehat{\boldsymbol{R}}_{\mathrm{cn}}$ is the estimated clutter covariance matrix. The suggested implementation does not require any additional resources when compared with the conventional implementation, but enables an easy and efficient mechanism for the filter order update. As shown in the numerical results section, the order update can be especially valuable when the number of auxiliary vectors varies from cell to cell, which is the case for the detection in non-stationary clutter with clutter discretes [27]. The suggested implementation can be relatively easily extended to the space–time adaptive processing systems having multiple transmitting and receiving antennas [28].

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